Specification and Estimation of the Transfer Function in Paleoclimatic Reconstructions *

Maximilian Auffhammer
University of California, Berkeley

Seung-Jick Yoo
Korea Energy Economics Institute

Brian Wright
University of California, Berkeley

June 1, 2009

Abstract

We identify two issues with a standard time series approach to reconstruction of past climate fluctuations from paleoclimatic data series, one related to specification of the estimated relationship between climate and the paleoclimatic index, the other to the methodology of estimation. We show that the standard approach provides biased estimates of the reconstructed climate series and underestimates the true variability of historical climate. We demonstrate that inversion of the estimated response function between tree ring growth and climate indicators provides consistent estimates of historical climate. The inversion method results in an overestimation of the variance. We show analytically as well as using Monte Carlo experiments and actual tree ring data, that use of the new specification and reconstruction procedure can be crucial for inferences about the nature of past climate and interpretation of recent climate variations.

Keywords: climate change, reconstruction method, inversion, paleoclimatic data, climate variability

*Preliminary draft not for attribution. Auffhammer: Associate Professor, Department of Agricultural and Resource Economics, 207 Giannini Hall, University of California, Berkeley, CA 94720-3310, USA, Phone: (510) 643-5472, Fax: (510) 643-8911, E-Mail: auffhammer@berkeley.edu; Yoo: Senior Research Fellow, Korea Energy Economics Institute, 665-1 Naeson-dong, Euiwang-si, Kyunggi-do, South Korea 437-713, Tel: +82-31-420-2226, Fax: +82-31-420-2162, E-mail: sjyoo@keei.re.kr; Wright: Professor, College of Natural Resources, Department of Agricultural and Resource Economics, 207 Giannini Hall #3310, Berkeley, CA 94720-3310, Tel: 510-642-9213, Fax: 510-643-3075, E-mail: wright@are.berkeley.edu

1. Introduction

The relation between global climate change and increased emissions of greenhouse gases is at the center of the debate on global warming. To investigate this relationship, a solid knowledge of the variability and the trend, if any, in the natural climate system as a baseline for climate behavior is essential. Unfortunately long time series of direct instrumental measures of global or local climate variables are unavailable. For example the longest instrumentally-measured temperature series are available only over about two centuries (Jones, Wigley and Wright 1986, Hansen and Lebedeff 1987, National Research Council 1998), during which the emissions of greenhouse gases from human economic activities have also risen continuously, and atmospheric carbon dioxide concentrations have increased.

To supplement the brief observed temperature series, the long run history of natural climate variability has been reconstructed using paleoclimatic data series including tree-ring index series, ice cores, pollen series, coral, and faunal and floral abundance in deep-sea cores. The paleoclimatic data series are selected for their length of sample span, sensitivity to climate variability, and relative lack of disturbance from non-climate factors. Examples of such reconstructions include Briffa, Bartholin, Eckstein, Jones, Karlén, Schweingruber and Zetterberg (1990), Briffa, Jones, Bartholin, Eckstein, Schweingruber, Karlén, Zetterberg and Eronen (1992), Briffa, Schweingruber, Jones, Osborn, Harris, Shiyatov, Vaganov and Grudd (1998), Briffa, Osborn, Schweingruber, Harris, Jones, Shiyatov and Vaganov (2001), Scuderi (1993), Hughes and Brown (1992), Bradley and Jones (1992), Mann (2002), Mann, Bradley and Hughes (1998) and others surveyed in Jones, Briffa, Barnett and Tett (1998) and Jones, Osborn and Briffa (2001).

The IPCC (2001) states that the increase in temperature in the 20th century is likely to have been the largest of any century during the past 1,000 years based upon the analyses of the proxy data for the Northern Hemisphere. Recently, attention to statistical issues in dendroclimatology has been intensified by the controversy initiated by McIntyre and McKitrick (2005), who raise questions about the validity of the relation between the aggregation of reconstructions from multiple tree growth indices and temperature underlying the "hockey stick graph" of global average temperature used by the IPCC (2001), which originated from Mann et al. (1998). This debate has focused new scrutiny on the statistical validity of extracting climate information from proxy variables closely related to the up-trending instrumental temperature series.

In this paper, we address issues unrelated to aggregation of multiple reconstructions and focus on reconstruction of climate at a single site using a single proxy time series. We identify two specific issues with standard reconstruction approaches, one of specification of the estimated relationship between climate and the paleoclimatic index, the other with the methodology of estimation, each of which can to contribute to the underestimation of variability in reconstructed climate. We provide a simple alternative methodology of reconstruction, which can be used to extend the climate measure backwards in time. To develop our argument and demonstrate the use of our methodology we focus on examples of reconstructions using tree-ring indices, identified in recent surveys as the most accurate proxies for annually-resolved climate reconstruction (Jones et al. 1998).

Studies in dendroclimatology specify the growth of a tree as limited by relevant environmental factors including temperature, precipitation, and atmospheric concentration of CO_2. In empirical estimates of the relation between "tree growth", typically measured by an index of radial growth and/or latewood density, and environmental factors, summarized by a climate index (hereafter "climate"), one or two lags of environmental variables are typically found to significantly affect the current growth of the tree. The "response function," relating tree growth to environmental variables, forms the basis of the reconstruction methodology. For the purposes of this paper we follow von Storch, Zorita, Jones, Dimitriev, Gonzalez-Rouco and Tett (2004) and assume that the index representing the paleoclimatic data is correctly constructed and unbiased and that the causal relationship between climate and the paleoclimatic index exists and is correctly specified using information from experts in the relevant fields.

Whereas the response function relates tree growth to climate, climate reconstruction seeks to generate the climate record from tree growth. To achieve this, a "transfer function" is specified relating a climate variable to a tree growth measure. Climate is usually specified as dependent on the current and a few years of future radial growth of a tree, where the number of years of "leads" is chosen to match the number of "lags" identified in the response relation. This transfer function is then estimated directly for the sample period with an instrumental climate record. The estimated relation is then used to reconstruct the climate series using the full historical sample of the tree growth index. This standard approach to reconstruction raises two issues related to the accuracy of the reconstruction.

First, the fact that current and lagged observations of the climate variable causally affect tree growth does not imply that current and future tree growth should appear in the inversion of this relation. As we show below, whether the leads or the lags of the environmental factors should be included as predictors in the actual transfer function for a given response function depends upon the relative size of the parameters in the response function. Even when leads should be included in the transfer function, the number of leads is always greater than the number of lags in the response function. Furthermore, when the inclusion of leads is justified, current tree growth, which is typically included, should be excluded from the estimating equation.

A second, distinct set of problems arises when the transfer function is estimated directly via a regression that specifies the temperature data series as the dependent variable, with tree growth as an independent variable. Direct estimation of the transfer function instead of estimation of the casual response function is a classical example of the reverse regression problem (see for example Klepper and Leamer (1984)), which causes the estimator to be biased in small as well as large samples. Bias of the estimator even in a large sample poses a critical limitation in prediction or in reconstruction of temperature series in dendroclimatology. von Storch et al. (2004) pointed out another well known aspect of the reverse regression problem for a static response function. These regression based methods applied in dendroclimatological reconstructions for a single proxy (e.g. tree rings) series underestimate the true variability of historical climate at that location, given the validity of the response function.

Finally, in addition to the bias in reconstructions, the misspecification of the transfer function used to reconstruct climate from the paleoclimatic record results in an overestimate of the order of the autoregressive process of the reconstructed temperature series. Seater (1993) shows that the order of the autoregressive process of the reconstructed fourteen-century Fennoscandia temperature series by Briffa et al. (1990) is 17 with no time trend. While empirically the estimated order of the AR process also depends on the selection criterion used, we present theoretical analysis and Monte Carlo evidence indicating that direct estimation of the misspecified transfer function will tend to result in such overestimation of the autoregressive process in many cases. We show, using the data from Briffa et al. (1992), that the standard regression approach overestimates the order of the true AR process by three. This aspect of reconstruction is of crucial importance, since overestimation of the order of the AR process of temperature underestimates the variability of the true temperature movement so that inferences about the significance of observed climate abnormalities in a particular period are unreliable.

In short, the commonly applied methods of making inferences about past climate variability from tree-ring data or other paleoclimatic data series are subject to problems that may crucially affect inferences about the nature of past climate and recent changes in climate, even if temperature is accurately measured in the period where direct instrumental measurement is available, the response function is correctly specified, and the explanatory power of the estimated response relation is good.

In the following section we review standard reconstruction procedures of climate history from paleoclimatic data series, taking the specification of the response function as given. Specification of the transfer function by formal inversion of the theoretical climate-growth relation, and statistical issues in direct estimation of the parameters of the transfer function, are investigated in Section 3. Section 4 contains a Monte Carlo experiment exploring the impact of these methodological issues in drawing inferences regarding the history of climate and the nature of its baseline time series behavior. This section concludes with the reconstruction of the climate history using the standard and inversion approaches based on the data employed in Briffa et al. (1992). Conclusions and implications follow in Section 5.

2. Reconstruction Historical Climate

Biological growth of trees, expressed in tree-ring width or density is reported by Fritts (1991) to be limited by operational environmental factors including temperature, precipitation and CO_2 content of the atmosphere. It has long been recognized that if climate has been limiting tree growth in a systematic way, and the relation has been stable for thousands of years, a very long history of tree growth might be employed to extend backward the relatively brief recorded history of climate.

The biological causal relation between tree-ring growth and environmental factors that are inputs to the growth process is known as the *response function*. The inverse function of the response function is the *transfer function*. As outlined below, dendroclimatologists often estimate the transfer function directly, with a simple specification based upon inferences from prior estimation of the response function. The typical reconstruction procedure entails estimation of the functional relation between tree rings and one or more climate variables. The resulting estimates obtained over a calibration period are verified using data in a verification period. In choosing the climatic variables to include as predictors (for example, temperature or precipitation), one approach investigates the significance of the impact of the current and lagged climatic variables on tree growth to select the order of the lags of climate variable. For example, in Scuderi (1993) up to two lagged years are significant, in which case the specification is

$$\mathrm{TR}_t = \beta_o \cdot \mathrm{Temp}_t + \beta_1 \cdot \mathrm{Temp}_{t-1} + \beta_2 \cdot \mathrm{Temp}_{t-2} + \varepsilon_t \qquad (1)$$

where TR_t is a tree-ring index, Temp_t is temperature index at date t, and ε_t is assumed to be i.i.d. $N(0, \sigma^2)$. The inclusion of the lagged temperature variable is interpreted as reflecting year-to-year persistence of the effect of climate variables upon the response of tree growth (LaMarche Jr 1974). In the standard approach, the order of the lags of the climatic variable chosen for the response function determines the number of leads of tree-ring or other dendroclimatological variables in the transfer function. The transfer function is estimated directly using a multiple regression method. For example, Scuderi (1993), based on inference from the above response function specification, chose as the transfer function:

$$\text{Temp}_t = \gamma_o \cdot \text{TR}_t + \gamma_2 \cdot \text{TR}_{t+1} + \gamma_3 \cdot \text{TR}_{t+2} + \eta_t \tag{2}$$

The climate history is then reconstructed from the dendroclimatological data using the estimated transfer function coefficients ($\hat{\gamma}_1$, $\hat{\gamma}_2$, $\hat{\gamma}_3$). This reconstruction is used to infer the time series properties of climate, including the dynamics of baseline variation, counts, and duration of climatic extremes such as severe droughts and heat waves. In the next section we will show how to obtain the transfer function implied by a given response function. Further we will explore the statistical consequences of directly estimating transfer function (a standard approach in paleoclimatology) versus estimating the response function and inverting it.

3. From Response to Transfer Function

3.1 Specification of the Transfer Function

In this section we illustrate the specification and estimation issues in a simple case in which the response function contains only one lag of the climate variable (e.g. Briffa et al. 1992). Let TR_t be the annual increment of tree-ring width and Temp_t temperature at date t. The regression equation of the response function of tree growth to the temperature variation is

$$\text{TR}_t = \beta_o \cdot \text{Temp}_t - \beta_1 \cdot \text{Temp}_{t-1} + \varepsilon_t = \beta_o (1 - \phi) \text{Temp}_t + \varepsilon_t \tag{3}$$

where $\phi = \frac{\beta_1}{\beta_o}$ and the lag operator L is defined as $L^p\text{Temp}_t = \text{Temp}_{t-p}$, $p \; \epsilon \; N$. The measurement error is assumed to be i.i.d., $\varepsilon \sim N(0, \sigma^2)$. Inversion of equation (3) will vary depending upon the ratio of the parameters, β_o and β_1. First consider the case where $|\phi| < 1$. Assuming that Temp_t is a bounded sequence, we can solve equation (3) for Temp_t as follows (Hamilton 1994):

$$\text{Temp}_t \;\; = \;\; (\beta_o (1 - \phi L))^{-1} (\text{TR}_t - \varepsilon_t) \tag{4}$$

$$\cong \frac{1}{\beta_o}\left(\mathrm{TR}_t + \phi\mathrm{TR}_{t-1} + \phi^2\mathrm{TR}_{t-1} + \phi^3\mathrm{TR}_{t-3}\ldots + \phi^m\mathrm{TR}_{t-m}\right) + \nu_t$$

where $\nu_t = -\frac{1}{\beta_o}\left(1 + \phi L + \phi^2 L^2 + \phi^3 L^3 \ldots + \phi^m L^m\right)\varepsilon_t$. Therefore even if TR_t is determined by the current and the first lagged value of Temp_t in the response function, the corresponding transfer function should be defined as the sum of the lags of the tree growth measures and of the errors in the estimated response function back to the distant past with geometrically decreasing weights, as in (5). Only then will it reflect the biological relation that specifies the direction of causality in the response function. If $|\phi| < 1$, the marginal effect of current temperature is larger than that of the previous year. The inversion shown above implies that Temp_t in the transfer function should not be regressed on the current value of TR_t and its first lead, TR_{t+1} as is the standard approach given the response relation (3), but on the current observation and on the previous observations of tree-ring growth. Direct estimation of the transfer function could be conducted on equation (5):

$$\mathrm{Temp}_t = \theta_o\mathrm{TR}_t + \theta_1\mathrm{TR}_{t-1} + \theta_2\mathrm{TR}_{t-2} + \ldots + \theta_m\mathrm{TR}_{t-m} + \eta_t \tag{5}$$

Estimation of this relation, given the null hypothesis specified in equation (3), should restrict the coefficients to satisfy the restrictions $\theta_o = \frac{1}{\beta_o}$, $\theta_p = \phi^p\theta_o$, $p = 1, 2, \ldots, m$. Even though there are $m+1$ parameters to be estimated in (5) the actual number of the independent parameters is only two, β_1 and $\phi = \frac{\beta_1}{\beta_o}$. Finally, errors of estimation in equation (5) are serially correlated with each other up to the $\mathrm{m}^t h$ order. If this is ignored, the estimators for θ are inefficient.

Now consider the case where $|\phi| > 1$. We can invert equation (3) using the lead operator, which is the inverse of the lag operator $L^{-p}\mathrm{Temp}_t = \mathrm{Temp}_{t+1}$, for $p > 0$. Equation (7) provides the transfer function from inverting (3) if $|\phi| > 1$ (Hamilton 1994).

$$\begin{aligned}
\mathrm{Temp}_t &= \left(\beta_o\left(1 - \phi L\right)\right)^{-1}\left(\mathrm{TR}_t - \varepsilon_t\right) \qquad (6)\\
&\cong -\frac{1}{\beta_o}\left(\phi^{-1}\mathrm{TR}_{t+1} + \phi^{-2}\mathrm{TR}_{t+2} + \phi^{-3}\mathrm{TR}_{t+3}\ldots + \phi^{-(m+1)}\mathrm{TR}_{t+m+1}\right) + \nu_t
\end{aligned}$$

where $\nu_t = -\frac{1}{\beta_o}\left(\phi^{-1}L^{-1} + \phi^{-2}L^{-2} + \phi^{-3}L^{-3}\ldots + \phi^{-(m+1)}L^{-(m+1)}\right)\varepsilon_t$. For this parameterization of the response function, where lagged climate has a larger impact on tree ring growth than current climate, the matching transfer function is one where current temperature is not a function of current tree growth, but is a weighted sum of leads of tree growth and the errors ε_t with geometrically declining weights. If this transfer function is estimated directly, the restrictions on the coefficients and the serial correlation issues are similar to the $|\phi| < 1$ case.

In dendroclimatology a common practice has been to estimate the transfer function directly (see for example Graumlich and Brubaker (1986); Briffa et al. (1990); Till and Guiot (1990); Graumlich (1991); Scuderi (1993)), often after estimating the response function as a first step. This approach ignores the fact that the transfer function is misspecified if the underlying response function is correctly specified. Assuming that climate is accurately measured by the climate index in the sample period, the transfer function relating a climate variable to tree growth should be generated by inverting the estimated response function. If tree growth is generally limited by the current and previous years' climatic conditions (LaMarche Jr 1974) the climate measure in a given year should be calculated as a function of either current and past tree growth if $|\phi| < 1$, or of future tree growth excluding current year tree growth if $|\phi| > 1$. The relevant specification of the transfer function therefore depends upon the relative size of the marginal effect of the current (β_o) and the previous year's (β_1) climate on current tree growth. Empirically, the weights of the lags/leads should be calculated from the estimated response function parameters.

If the order of lags in temperature effects on current tree growth is not known *a priori*, the order of lags of the temperature series can be estimated *ex post* and utilized in inverting the response function into the transfer function, as illustrated above for the one-lag case for $|\phi| < 1$ or $|\phi| > 1$. In the following section we discuss statistical problems that can arise in direct estimation of the transfer function, and their effects on inferences about the long-run climate history.

3.2 Statistical Properties of Estimators of the Transfer Function

Statistical problems in reconstructing the history of climate variability can arise if the transfer function is misspecified, and also when a correctly specified transfer function is directly estimated instead of being inverted from the estimated response function. Here we focus on the latter problem, assuming correct specification. Direct estimation of the transfer function encounters the classical problem of errors in variables (Klepper and Leamer 1984). Ordinary Least Squares (OLS) estimators are biased and inconsistent if the tree-ring index series as predictors in the transfer function are subject to measurement errors due to disturbances (ε_t) in the response function. The estimates of the parameters of this direct estimation of the transfer function are biased downward not only in levels but also in variability. To demonstrate this, consider the simplest case below, which omits the issue of lags and leads, but can be easily extended to the general case. Specify the response, TR_t, as determined by the contemporaneous impact of Temp_t with an unobservable factor $\varepsilon_t \sim N(0, \sigma^2)$:

$$\mathrm{TR}_t = \beta \mathrm{Temp}_t + \varepsilon_t \tag{7}$$

As is standard in the literature, both TR_t and Temp_t are standardized to have mean 0 and standard deviation 1. In equation (7) this implies a coefficient of determination (R^2) of β^2. Given the standardization the OLS estimate of β is:

$$\hat{\beta} = \frac{\sum \mathrm{TR}_t \cdot \mathrm{Temp}_t}{\mathrm{Temp}_t^2} = \mathrm{cov}(\mathrm{TR}_t, \mathrm{Temp}_t) \tag{8}$$

The specification of the reverse regression used to reconstruct Temp_t's from TR_t's is

$$\mathrm{Temp}_t = \gamma \mathrm{TR}_t + \eta_t \tag{9}$$

Given the standardization again, the OLS estimate of γ is given by:

$$\hat{\gamma} = \frac{\sum \mathrm{Tr}_t \cdot \mathrm{Temp}_t}{\mathrm{TR}_t^2} = \mathrm{cov}(\mathrm{TR}_t, \mathrm{Temp}_t) \tag{10}$$

The OLS estimate, $\hat{\gamma}$, of the reverse equation equals the OLS estimate of the response function $\hat{\beta}$, because of the standardization of TR_t's and Temp_t's. Given the causal relation in (7)), the matching transfer function to be used for reconstruction is given by:

$$\text{Temp}_t = \frac{1}{\beta}(\text{TR}_t - \varepsilon_t) = \frac{1}{\beta}\text{TR}_t + \eta_t \tag{11}$$

noting that $\eta_t = -\frac{1}{\beta}\varepsilon_t$. If we take the ratio of the coefficients on the tree ring index from the reverse regression in (9) and the inversion method in (11) and notice that $|\beta| < 1$, we get:

$$\frac{\hat{\beta}}{\frac{1}{\hat{\gamma}}} = \frac{\hat{\gamma}}{\frac{1}{\hat{\gamma}}} = \hat{\gamma}^2 \ll 1 \tag{12}$$

Thus the OLS estimate of the coefficient of TR_t in the reverse regression is biased toward zero by a proportion $(1 - \beta^2)$, which is proportional to the coefficient of determination of the OLS estimate of the response function. This implies that the bias grows as the fit of the response function deteriorates.

Further, the reverse regression is subject not only to bias of the estimated coefficient but also to underestimation of the variance of the reconstructions $\left(\widehat{\text{Temp}}_t^r\right)$:

$$\text{var}\left(\widehat{\text{Temp}}_t^r \gamma\right) = \gamma^2 \text{var}(\text{TR}_t) = \gamma^2\beta^2 \text{var}(\text{Temp}_t) + \gamma^2 \text{var}(\varepsilon_t) = \gamma^2 \tag{13}$$

since $\text{var}(\varepsilon_t) = 1 - R^2 = 1 - \gamma^2$. The variability of the reconstruction by reverse regression is determined by the estimate of the coefficient, γ , which as shown above equals β in expectation. Hence the variability of the reconstruction is also biased downward and decreasing in the fit of the response function. This is consistent with the observation made in von Storch et al. (2004). On the other hand, if the transfer function is obtained by inversion of the estimated response function, the variance of the reconstruction,$\widehat{\text{Temp}}_t^i$, in expectation is overestimated relative to the true (unobserved) climate:

$$\text{var}\left(\widehat{\text{Temp}}_t^i|\beta\right) = \text{var}\left(\frac{1}{\beta}\text{TR}_t\right) = \frac{1}{\beta^2}\text{var}(\text{Temp}_t) = \text{var}(\text{Temp}_t) + \frac{1}{\beta^2}\text{var}(\varepsilon_t) = \frac{1}{\beta^2} > 1 \tag{14}$$

The overestimation of the true variance of the climate series grows inversely with the fit of the response function. A noisy proxy therefore leads to a noisy reconstruction.

The autocorrelation order of long run history of the reconstructed climate series is another important characteristic in making inferences about its baseline fluctuation. When the dependent variable (tree ring index) is caused by the lagged and the current effects of an explanatory variable (climate variable), the dependent variable exhibits time dependent behavior. Let us consider the implications for the autoregressive order of the estimated climate series from the two estimation approaches using the following simple response function as an example:

$$\mathrm{TR}_t = \beta_o \mathrm{Temp}_t + \beta_1 \mathrm{Temp}_{t-1} + \varepsilon_t \tag{15}$$

where both TR_t and Temp_t are standardized and $\varepsilon_t \sim N(0, \sigma^2)$. Following the practice commonly followed in the reconstruction literature, the transfer function below is estimated directly:

$$\mathrm{Temp}_t = \gamma_o \mathrm{TR}_t + \gamma_1 \mathrm{TR}_{t+1} + \eta_t \tag{16}$$

Since the variables are standardized we can represent the OLS estimates in (16) as a function of the OLS estimates in (15).

$$\begin{aligned} \hat{\gamma}_o &= \frac{\hat{\beta}_o \left(1 - \hat{\beta}_1^2\right)}{1 - \hat{\beta}_o^2 \hat{\beta}_1^2} \\ \hat{\gamma}_1 &= \frac{\hat{\beta}_1 \left(1 - \hat{\beta}_o^2\right)}{1 - \hat{\beta}_o^2 \hat{\beta}_1^2} \end{aligned} \tag{17}$$

Then the reconstruction of the climate series using reverse regression, $\widehat{\mathrm{Temp}}_t^r$, is given by $\widehat{\mathrm{Temp}}_t^r = \hat{\gamma}_o \mathrm{TR}_t + \hat{\gamma}_1 \mathrm{TR}_{t+1}$ To focus on the problem caused by estimating a misspecified transfer function, abstracting from the errors-in-variables problem discussed above, we remove the unobserved factor, ε_t, from TR_t , resulting in a perfect fit for the response function. Recognizing that $\hat{\beta}_i$ is an unbiased estimate of β_i for $i = 0, 1$, and using the true relation between β's and γ's, we have:

$$\begin{aligned} \widehat{\text{Temp}}_t^r &= \frac{1}{1-\beta_o^2\beta_1^2}\left[\beta_o^2\left(1-\beta_1^2\right)+\beta_1^2\left(1-\beta_1^2\right)\right]\text{Temp}_t \\ &+\frac{1}{1-\beta_o^2\beta_1^2}\left[\beta_o\beta_1\left(1-\beta_1^2\right)\text{Temp}_{t-1}+\beta_o\beta_1\left(1-\beta_o^2\right)\text{Temp}_{t+1}\right] \end{aligned} \quad (18)$$

Reconstructed $\widehat{\text{Temp}}_t^r$'s are the weighted sum of the lag and lead of Temp_t's as well as the current value. Reconstructions show dependency across time even if they are reconstructed from error-free variables. This is due to the misspecified transfer function, which does not reflect the physical causal relationship given by the response function. On the other hand the reconstruction by inversion of the correct specification as in (7) will only exactly match the true Temp_t if TR_t is free from errors, ε_t, as assumed in this illustration. When there are errors in the response function, the correct inversion also suffers the risk of introducing time dependent behavior in the reconstructed series - even if the errors, ε, are i.i.d.

In summary, a noisier response function will result in noisier reconstructions, with potential time dependence, even if the response function is correctly inverted. But the estimation of a misspecified transfer function induces time dependency not only due to the errors in the response relation but also due to misspecification of the transfer function. The limitations of this method of reconstruction do not disappear if the fit of the response relation is perfect. For high-quality data, for which the response relation has high explanatory power, the inversion method is clearly superior. For low-quality data, little can be expected from either method.

3.3 Monte Carlo Experiments and Estimation

To more fully investigate the issues of specification estimation addressed above, we conducted several Monte Carlo experiments as well as a reconstruction using an actual tree ring index (Briffa et al. 1992).[1]

[1] The Matlab code for all Monte Carlo exercise is available from the authors upon request.

3.3.1 Homogeneous Series

In the first experiment we chose β_o and β_1 so that their ratio is equal to the ratio of the values found by fitting the tree ring series to the temperature series (i.e., estimating the response function) (Briffa et al. 1992). A set of independent Temp_t's is generated from the standard normal distribution and held constant for the experiment. The i.i.d. disturbances, ε_t , were repeatedly drawn from a mean zero normal with the variance chosen so that the variance of the TR_t's is 1 given the values of β_o and β_1. For each experiment we chose the sample size $T = 1000$, and we replicated the sample 1,000 times.

To apply the inversion method, we estimated the response function (15) for the last 100-year subsample, and reconstructed the series Temp_t for the whole sample from the series for TR_t via equation (5) using $m = 10$. The reverse regression method was implemented by estimating the transfer function (16) from the last 100-year subsample, and reconstructing the whole sample from this estimated regression relation. As an initial comparison of the two methods of reconstruction we take the average of the reconstructions for each year of each of the 1,000 experiments. The difference of the average reconstruction from the true Temp_t is plotted in Figure 1 over the whole sample period.

On average, deviations of the values produced by reconstruction by inversion of the estimated response function from the true generated time series of Temp_t's are dense around 0, indicating reconstruction by the inversion method is quite accurate. On the other hand, the direct estimation of the transfer function, as given by equation (16), results in a much noisier estimate of the true mean reflected in the larger variance of the series plotted in figure 1.

To further compare the performance of the two reconstruction methods, we perform additional Monte Carlo experiments for different sets of parameter values for β_o and β_1 . For the first set of experiments we hold β_o fixed and explore the performance of the two reconstruction methods for the valid range of β_1, given that the Temp_t's and TR_t's are standardized. As a first measure of performance, we use the mean absolute deviation of reconstructions from the generated "true" value for each of the two reconstruction methods, which is defined as

$$\text{MAD} = \frac{\sum_{t=1}^{T} |f_t - \text{Temp}_t|}{T} \tag{19}$$

where f_t is the average of the reconstructions from 1,000 replications. The quality of the reconstruction of Temp_t depends on two factors, the explanatory power of Temp_t in the response function and the ratio of the current impact of climate to its lagged impact. Figure 2 shows the MAD performance of the two methods for three different levels of β_o across the valid range of β_1. Reconstruction either by inversion of the estimated response function or by estimation of the transfer function improves as the explanatory power of the climate variable in the response function increases. If the respondent variable is severely contaminated by unidentifiable random effects, high quality reconstructions of climate variables from the respondent variable are not possible with either method.

Irrespective of the explanatory power of climate variables in the response function, the inversion method proves superior in reproducing the true Temp_t's. As the ratio of β_o to β_1 of the response function increases, the relative role of the lagged explanatory variables decreases and the accuracy of the reconstructions by inversion improves. The average of the mean absolute deviation from the true values of reconstructions by reverse regression is almost 10 times higher than that of the inversion method even when the current effect of Temp_t on TR_t compared to the lagged impact is dominant, for example $\beta_o = 0.9$, $\beta_1 = 0.01$. Reverse regression bias makes the reconstruction less accurate.

Table (1) shows results from Mote Carlo experiments of reconstructions for seven combinations of (β_o, β_1) . For each experiment, the true standard deviation of Temp_t is exactly 1 for the sample period. The top row in the table indicates the magnitude of the coefficients of the response function. The share of the variance in temperature coming from a signal instead of noise (ε_t) increases as we go to the right in the table, as indicated by the implicit variance of ε_t.

Consistent with figure (2), the table indicates that the MAD of the reconstructions using the inverse regression methods is consistently smaller than that of the reverse regression method and converges towards zero as the response function becomes less noisy. The reverse regression reconstructions never become unbiased. The mean of the estimated standard deviation (S.D.) of reconstruction is biased upwards for the inversion approach and biased downwards for the reverse regression approach. When the explanatory power of Temp_t's is 0.99 as in the case with (β_o, β_1) equal to (0.8, 0.59) or (0.9, 0.42), the means of the estimated standard deviation for the reconstructions from reverse regression ($\widehat{\text{Temp}}_t^r$) are 0.85 and 0.92 while the reconstructions from the inversion method ($\widehat{\text{Temp}}_t^i$) have an estimated standard deviation of 1.04 and 1.03 respectively, which is close to the true value of 1. The bias in variance reduces dramatically going from left to right as the explanatory power of the Temp_t's increases (as $\text{var}(\varepsilon_t)$ goes to 0.01) and almost disappears for the inverse regression method, but not for the reverse regression method. The latter technique's bias, given a valid response function with high explanatory power, is predominantly due to misspecification of the transfer function - not to the error in TR_t. It should be emphasized that the deviations of the variance of reconstructions in the inversion approach are not due to a problem with the method, but are an inevitable consequence of the low explanatory power of climate in the determination of tree growth.

Since MAD is only one measure of performance, we calculate the mean "Reduction of Eror" (RE) and the "Coefficient of Efficiency", which range from $-\infty$ to 1. Both of the measures are standard in the dendroclimatology literature and described in detail in Cook, Briffa and Jones (1994). For each statistic, a value of zero indicates that the reconstruction method is equivalent in performance to using the in sample mean. Values less than one indicate that the mean outperforms the reconstruction. Values greater than zero and less than one indicate predictive ability of the model over simply using the mean. As table 1 indicates, the inverse regression method for both statistics always outperforms the mean. Both statistics approach the theoretical maximum of 1 for the last two low noise scenarios. The reverse regression method has negative values for both statistics for the first two scenarios. Further, neither the RE nor the CE are greater than those of the inverse regression method for any of the considered coefficients.

The bottom 6 rows of table 1 show supporting Monte Carlo evidence of our theoretical claim from the previous section, that the reverse regression method will overestimate the order of the autoregressive process. In the experiments above we have generated Temp_t's as independent processes over time, which implies that Temp_t is AR(0). For each iteration and reconstruction method, we use the Schwarz criterion to render a consistent estimate of the order of the process. (Note that using the inconsistent AIC would further increase the estimated order of the autoregressive process.) As above, the success of each method in correctly identifying the order of the autoregressive process depends on the explanatory power of the series Temp_t in determining TR_t. When explanatory power is very low, neither method is successful at correctly identifying the correct order. When the variances of errors, ε_t , are less than 0.2, the reconstructions by inversion of the response function are successfully identified as independent processes over time most of the time. For the response function, which contains the least degree of noise, the inverse regression method correctly identifies the process as AR(0) 100% of the time, while the reverse regression methods predict the order to be higher than AR(5) 100% of the time.

The reconstructions by reverse regression yield consistently erroneous, positive orders of the autoregressive process. This is one likely reason that Seater (1993) identified the underlying process of the reconstructed temperature series of Briffa et al. (1992) as AR(17). The second column which is the Monte Carlo using parameters whose ratio is equal to that of those estimated in Briffa et al. (1992), show that the reconstructions indicate an AR(5) or higher order AR process in 14% of the cases even though the true process is AR(0). von Storch et al. (2004) have pointed out this underestimation of the variability of the climate reconstructions, looking at studies using principal components of the proxy data series as predictors of the instrumental temperature series. Above we formalized this notion and showed an additional methodological issue contributing to the underestimate of variability in the reconstruction.

3.3.2 Heterogeneous Series

In the previous Monte Carlo experiment we assumed that climate follows a homogeneous time series process e.g. AR(0). It is theoretically possible that the history of temperature/climate includes structural changes. Ideally, the method of reconstruction should reproduce the heterogeneity in the time series characteristics, if it is present. In this section, we generate the Temp_t series as a combination of subseries that follow different time series processes. We generate a 1,000-year sample period, starting with 200 years of AR(0), followed by 200 years of AR(1), followed by 300 years of AR(0), followed by 100 years of AR(2) and the most recent 200 years following an AR(0) process. The last one hundred years, which are constructed as AR(0) will be used to estimate the relation between Temp_t's and TR_t's using the inverse and reverse regression method. We then reconstruct the full 1000 year history using both methods and test the order of the autoregressive process for the 5 segments from each reconstruction. We repeat this exercise 1000 times and record the number of times each reconstruction correctly identifies the order of the autoregressive process.

Table (2) shows the results from this experiment. The inversion method correctly identified the order of the original temperature series from the reconstruction in all the cases for the AR(0) process, 99% of the cases for the AR(2) process and 97.2% of the time for the AR(1) process. For the in sample years, the inverse regression method provided reconstructions which were correctly identified as AR(0) 100% of the time. The reverse regression method overestimated the order of the autoregressive process *in all cases shown.* This is consistent with the theoretical result in equation (19).

The results in figure (3) are striking. They highlight the contrast between the performance of the two methods of reconstruction in replication of the true underlying data generating process. They also show the risk of making incorrect inferences about the time series properties of the baseline movement when reconstructions are made from the reverse regression method.

3.3.3 Estimation and Reconstruction

To provide a concrete example of the consequences from reconstructing a climate index using a proxy index, we estimate the response function using the 1876-1974 temperature and latewood density index used by Briffa et al. (1992), and reconstruct the climate series for a 1524 year time span. For the reverse regression we replicate the results in the first model given in table 2 in Briffa et al. (1992) using one lead of the tree ring and maximum latewood density index. For the inverse regression method we only use the maximum latewood density index with one lag, which is significant in the response function using Newey and West (1994) standard errors. The fit of the response function in this example is very good for this given data set (similar to the 4^{th} and 5^{th} example in table 1). The results from the two reconstruction methods are summarized in Table 3. Figure 3 plots the actual temperature and reconstructions using both methods.

Matching the results from the Monte Carlo experiment, the estimated standard deviation of the reconstructions from the inversion method is 1.45 times the true standard deviation, thus overestimating the climate variability. The reconstruction via the inversion method results in a reconstruction with an autoregressive process matching the order of original series - AR(0). On the other hand, estimation of the reverse regression underestimates the standard deviation by 29%, resulting in an overly smooth series. It follows an AR(3) process, not matching the order of the actual temperature series. These results provide supporting evidence of what von Storch et al. (2004) noted - namely that traditional reconstruction methods result in an overly smooth historical climate record. In the case considered here this excessive smoothing is a consequence of the statistical estimation issues pointed out above as well as the misspecification of the transfer function. We further conducted calibration/verification exercise using the year 1925-1976 as a calibration period and the year 1876-1924 as a verification period. As table (3) shows, the inverse regression method outperforms the reverse regression method using the commonly used RE and CE measures of fit we have also employed earlier.

4. Conclusion

In this paper we investigate the adequacy of the commonly employed reconstruction method of past climate fluctuations from paleoclimatic data series such as tree-ring index series, ice cores, pollen series, assuming that the response function relating the chosen paleoclimatic index to the climate measure is correctly specified. We identify two problems with the standard approaches, one of specification of the estimated relationship between climate and the paleoclimatic index, the other with the estimation methodology. We demonstrate that reconstruction from the correctly specified inversion of the estimated response function is strongly preferred, from a statistical point of view, to direct estimation of a transfer function relating the climate index to a paleoclimatic index.

We show that the specification of the transfer function should be determined by the specification of the response function. Whether leads and/or lags of the environmental factors should be included as predictors in the transfer function depends on the relative size of the parameters of the response function. Whether or not the transfer function is correctly specified, direct estimation of the transfer function is a classic example of the reverse regression problem, which causes the estimators to be biased in large as well as small samples.

Further, we have explored the underlying causes of the issue raised by von Storch et al. (2004): The estimated degree of variation of reconstructed climate, which is central to the climate change debate, is underestimated as measured by the standard deviation of the reconstructions. Further, the method proposed in this study improves the accuracy of climate reconstruction found in the literature (Briffa et al. 1992) on average by an order of 3 or 4, measured by mean absolute deviations, in our Monte Carlo experiment.

In addition, the misspecification of the transfer function results in overestimation of the order of the autoregressive process of the reconstructed series. The reconstructions by the direct estimation of a misspecified transfer function yield consistently erroneous and positive orders of the autoregressive process in Monte Carlo experiments and the empirical example. As a result of misspecification of the transfer function, and the bias induced by the reverse regression estimation procedure, fluctuations of the reconstructed climate series are underestimated. Inferences from such series regarding the existence of abnormalities in particular periods, including the most recent period, are unreliable.

We also demonstrate when the underlying data generating process of the climate series is subject to change, direct estimation of the transfer function fails to reconstruct the change in the underlying data generating process or to detect the heterogeneity of the underlying process in the long history. This constitutes a significant limitation in making comparisons of the underlying process between different periods.

The quality of the paleoclimatic data series, the specification of the response relation, and its explanatory power, are of course crucial issues in reconstruction of the history of climate change. What we show here is that even if the data are excellent, and the response relation is correctly specified and has high explanatory power, direct estimation of a correctly or incorrectly specified transfer function can produce highly unreliable information about the history of climate. The inversion of the estimated response function is the preferred method for reconstruction of climate history. It generally generates more reliable information, given the quality of the available data and the specification of the response relation. An interesting extension of this work would explore the consequences of omitted variables in the response function.

References

Bradley, R. and Jones, P.: 1992, *Climate Since AD 1500*, Routledge, London.

Briffa, K., Bartholin, T., Eckstein, D., Jones, P., Karlén, W., Schweingruber, F. and Zetterberg, P.: 1990, A 1,400-year tree-ring record of summer temperatures in Fennoscandia, *Nature* **346**(6283), 434–439.

Briffa, K., Jones, P., Bartholin, T., Eckstein, D., Schweingruber, F., Karlén, W., Zetterberg, P. and Eronen, M.: 1992, Fennoscandian summers from ad 500: temperature changes on short and long timescales, *Climate Dynamics* **7**(3), 111–119.

Briffa, K., Osborn, T., Schweingruber, F., Harris, I., Jones, P., Shiyatov, S. and Vaganov, E.: 2001, Low-frequency temperature variations from a northern tree ring density network, *Journal of Geophysical Research* **106**(D3), 2929–2942.

Briffa, K., Schweingruber, F., Jones, P., Osborn, T., Harris, I., Shiyatov, S., Vaganov, E. and Grudd, H.: 1998, Trees tell of past climates: but are they speaking less clearly today, *Phil Trans R Soc Lond B* **353**, 65–73.

Cook, E. R., Briffa, K. R. and Jones, P. D.: 1994, Spatial regression methods in dendroclimatology: A review and comparison of two techniques, *International Journal of Climatology* **14**(4), 379–402.

Fritts, H.: 1991, *Reconstructing Large-scale Climatic Patterns from Tree-ring Data: A Diagnostic Analysis*, University of Arizona Press, Tuscon.

Graumlich, L.: 1991, Subalpine Tree Growth, Climate, and Increasing CO_2: An Assessment of Recent Growth Trends, *Ecology* **72**(1), 1–11.

Graumlich, L. and Brubaker, L.: 1986, Reconstruction of annual temperature(1590-1979) for Longmire, Washington, derived from tree rings., *Quaternary Research* **25**(2), 223–234.

Hamilton, J.: 1994, *Time series analysis*, Princeton University Press Princeton, NJ.

Hansen, J. and Lebedeff, S.: 1987, Global trends of measured surface air temperature, *Journal of Geophysical Research* **92**(D11), 13345–13372.

Hughes, M. and Brown, P.: 1992, Drought frequency in central California since 101 BC recorded in giant sequoia tree rings, *Climate Dynamics* **6**(3), 161–167.

IPCC: 2001, *Climate Change 2001: The Scientific Basis*, Cambridge University Press, Cambridge.

Jones, P., Briffa, K., Barnett, T. and Tett, S.: 1998, High-resolution palaeoclimatic records for the last millennium: interpretation, integration and comparison with General Circulation Model control-run temperatures, *The Holocene* **8**(4), 455.

Jones, P. D., Osborn, T. J. and Briffa, K. R.: 2001, The Evolution of Climate Over the Last Millennium, *Science* **292**(5517), 662–667.

Jones, P., Wigley, T. and Wright, P.: 1986, Global temperature variations between 1861 and 1984, *Nature* **322**(6078), 430–434.

Klepper, S. and Leamer, E.: 1984, Consistent Sets of Estimates for Regressions with Errors in All Variables, *Econometrica* **52**(1), 163–184.

LaMarche Jr, V.: 1974, Paleoclimatic Inferences from Long Tree-Ring Records: Intersite comparison shows climatic anomalies that may be linked to features of the general circulation, *Science* **183**(4129), 1043.

Mann, M., Bradley, R. and Hughes, M.: 1998, Global-scale temperature patterns and climate forcing over the past six centuries, *Nature* **392**, 779–787.

Mann, M. E.: 2002, The Value of Multiple Proxies, *Science* **297**(5586), 1481–1482.

McIntyre, S. and McKitrick, R.: 2005, Hockey sticks, principal components, and spurious significance, *Geophysical Research Letters* **32**(3), L03710.

National Research Council: 1998, *Decade-to-Century-Scale Climate Variability and Change: A Science Strategy*, National Academies Press, Washington, D.C.

Newey, W. and West, K.: 1994, Automatic Lag Selection in Covariance Matrix Estimation, *The Review of Economic Studies* **61**(4), 631–653.

Scuderi, L. A.: 1993, A 2000-Year Tree Ring Record of Annual Temperatures in the Sierra Nevada Mountains, *Science* **259**(5100), 1433–1436.

Seater, J.: 1993, World Temperature-Trend Uncertainties and Their Implications for Economic Policy, *Journal of Business & Economic Statistics* **11**(3), 265–277.

Till, C. and Guiot, J.: 1990, Reconstruction of precipitation in Morocco since 1100 AD based on Cedrus atlantica tree-ring widths, *Quaternary Research* **33**, 337–351.

von Storch, H., Zorita, E., Jones, J. M., Dimitriev, Y., Gonzalez-Rouco, F. and Tett, S. F. B.: 2004, Reconstructing Past Climate from Noisy Data, *Science* **306**(5696), 679–682.

Table I: Summary of Monte Carlo Experiment (T=1,000; # of iteration =1,000; Inv: Inversion Method; Rev: Reverse Regression Method)

	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
	0.4	0.1	0.4	0.2	0.6	0.2	0.8	0.2	0.9	0.1	0.8	0.59	0.9	0.42
var(e)	0.83		0.8		0.6		0.32		0.18		0.0119		0.0136	
	Inv.	Rev.	Inv.	Rev.	Inv.	Rev.	Inv.	Rev.	Inv.	Rev.	Inv.	Rev.	Inv.	Rev.
MAD	0.10	0.66	0.12	0.65	0.04	0.50	0.02	0.30	0.01	0.16	0.03	0.44	0.00	0.34
SD	2.84	0.47	3.13	0.49	1.75	0.67	1.26	0.85	1.13	0.94	1.04	0.85	1.03	0.92
RE	0.15	-3.39	0.12	-2.79	0.35	-0.45	0.66	0.50	0.82	0.78	0.96	0.57	0.98	0.78
CE	0.15	-3.42	0.12	-2.81	0.35	-0.46	0.66	0.49	0.82	0.78	0.96	0.56	0.98	0.78
AR(0)	19%	18%	5%	2%	15%	1%	53%	0%	93%	28%	100%	0%	100%	0%
AR(1)	80%	46%	87%	23%	78%	59%	47%	95%	7%	72%	0%	0%	0%	0%
AR(2)	1%	20%	9%	31%	7%	30%	1%	5%	0%	0%	0%	0%	0%	0%
AR(3)	0%	9%	0%	16%	0%	7%	0%	0%	0%	0%	0%	0%	0%	20%
AR(4)	0%	4%	0%	14%	0%	3%	0%	0%	0%	0%	0%	0%	0%	80%
AR(5+)	0%	3%	0%	14%	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%

Table II: Distribution of Estimated Autoregressive Order of Reconstructions for Heterogeneous Temperature Process By Method

Truth	Method	AR(0)	AR(1)	AR(2)	AR(3)
AR(1)	Inverse		97.20%	0.50%	2.30%
	Reverse			100%	
AR(2)	Inverse		1.00%	99.00%	
	Reverse				100%
AR(0)	Inverse	100%			
	Reverse		29.50%	70.50%	
AR(0) (In-Sample)	Inverse	100%			
	Reverse		97.50%	2.50%	

Table III: Comparison of Reconstruction in-sample period using Fennoscandia (Briffa et al., 1992)

	Actual Temperature	Reconstruction by Inversion Method	Reconstruction by Reverse Regression
Average	0	0	0
Standard deviation	1.01	1.46	0.71
Order of AR process	0	0	3
Correlation with Temperature	1	0.71	0.72
RE**	–	0.52	0.2
CE**	–	0.52	0.21

Note: We used the years 1925-1976 as a calibration period and 1876-1924 as the verification period. RE and CE are calculated using the formulas given in Mann and Rutherford (2002) with N=101.

Figure 1: Difference of the Mean Annual Reconstruction from the True Temperature ($\beta_o = 0.68$; $\beta_1 = 0.10$)

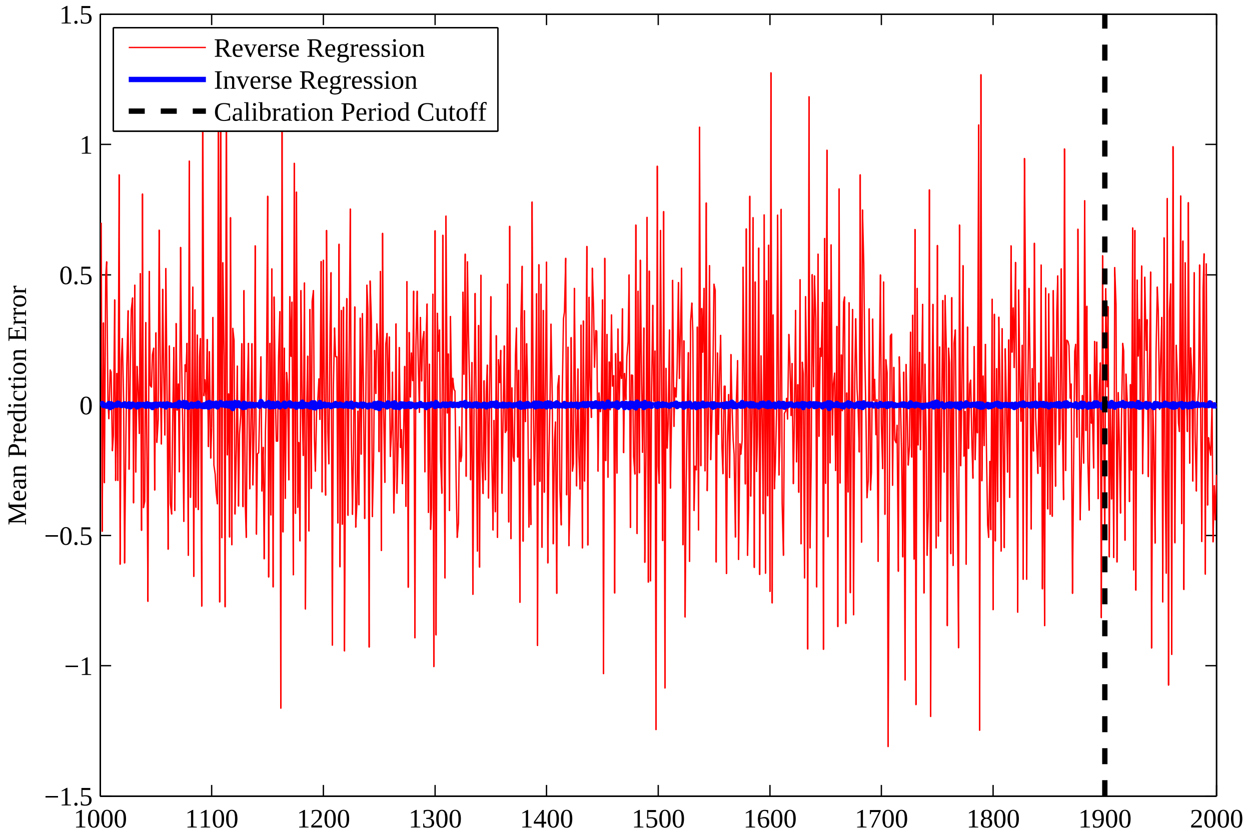

Figure 2: Mean Absolute Deviations of Reconstructions

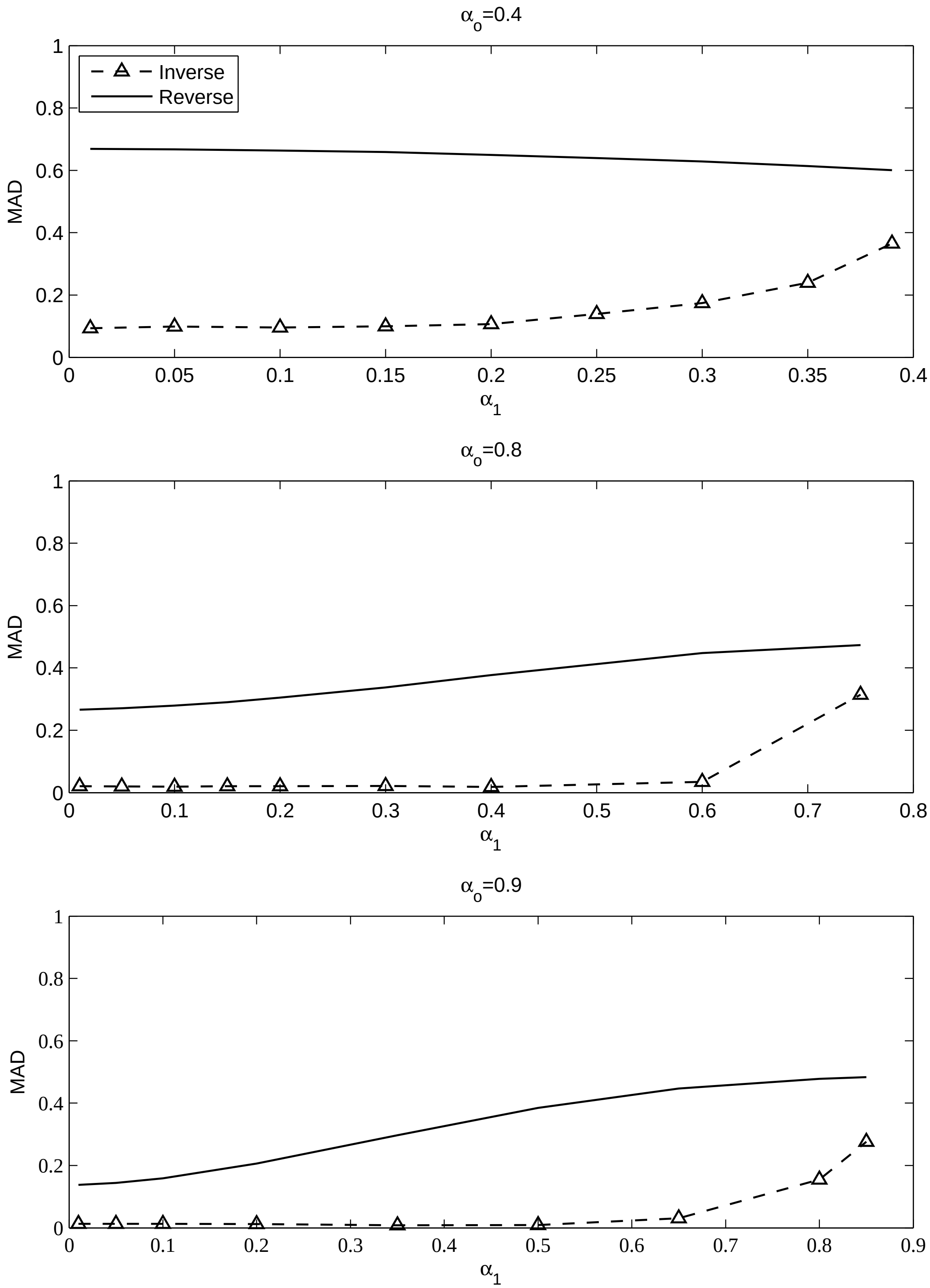

Figure 3: Reconstruction of the Observed Temperature Record in Briffa et al (1992)