On the Identification of DSGE Models

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June 1, 2009

Abstract

This paper provides conditions for identifying the parameters of a DSGE model and presents its reduced form in the general case when the number of shocks does not equal the number of observed endogenous variables. Combining results from classical econometric theory with structural identification analysis in control theory, we establish an upper bound on the number of free parameters that can be estimated, and by implication, the minimum number of parameters that must be held fixed. The assumption that shocks are univariate instead of vector-autoregressive processes often impose enough restrictions to satisfy the order condition. A rank condition is also developed to check whether there is a one-to-one mapping from the parameters of the optimizing model to the reduced form that induces the autocovariances. We show that identification can fail even in a simple stochastic growth model. Our conditions do not depend on the choice of the estimator and should be verified before estimation.

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The second author acknowledges financial support from the National Science Foundation (SES-0549978).

1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have now reached the level of sophistication to permit analysis of important policy and theoretical macroeconomic issues. Whereas the parameters in these models used to be calibrated, numerical advances in the last two decades have made it possible to estimate models with as many as a hundred parameters. While researchers are aware that not all model parameters can be estimated, a procedure has yet to exist that tells us in a systematic manner how many parameters are identifiable. This paper establishes the necessary (order) and sufficient (rank) conditions for identification in DSGE models. We show how to determine the maximum number of parameters that can be identified, and thus establishes the minimum number of parameters that needs to be fixed.

In a typical DSGE analysis, one starts by writing down an optimizing model expressed in terms of the so-called deep parameters. One then log-linearizes the model that is now expressed in terms of the structural parameters. The structural model is 'solved', and the model solution is expressed in terms of the parameters, Λ. The solution is then put to a state space form which is used to set up the likelihood for estimation. These steps are routine in DSGE analysis. Rarely discussed is the fact that the solution equations can be simplified to remove identities and linearly dependent relations. We will henceforth refer to the resulting set of equations as the reduced form, whose parameters will be denoted Π. Let Θ be the parameter of interest, which we take to be the deep parameters. Identification can fail when Π is uninformative about θ. This can arise if Π is uninformative about Λ, and/or Λ is uninformative about θ. As Π ultimately determines the information about the model that can be recovered, we provide a complete characterization of the reduced form, given the log-linearized solution to the DSGE model. We generalize results of Ravenna (2007) to allow the number of shocks to be different from the number of endogenous state variables in the model.

While the solution of DSGE models is a system of linear simultaneous equations, rank and order conditions developed for identification of static models with i.i.d. shocks are no longer adequate when the shocks are serially correlated because the class of observationally equivalent models is broader when dynamics are involved. More precisely, the linear static models $\Gamma_0 y_t = \Gamma_1 x_t + e_t$ and $\tilde{\Gamma}_0 y_t = \tilde{\Gamma}_1 x_t + \tilde{e}_t$ are observationally equivalent if there exists a matrix F such that $\tilde{\Gamma}_0 = F\Gamma_0, \tilde{\Gamma}_1 = F\Gamma_1$, and $\tilde{e}_t = Fe_t$. In a dynamic setting, the lag structure plays a role. Two linear models $\Gamma_0(L)y_t = \Gamma_1(L)x_t + \Gamma_2(L)e_t$ and $\tilde{\Gamma}_0(L)y_t = \tilde{\Gamma}_1(L)x_t + \tilde{\Gamma}_2(L)e_t$ are observationally equivalent if they have the same autocovariance structure (or the same spectrum). Unlike static models, fixing $F = \Gamma_2(0)$ is not enough for identification. Deistler (1976) and Hatanaka (1975) analyzed linear dynamic models in situations when the dimension of the shocks equals the number of endogenous variables. This is generally not the case for DSGE models which are intrinsically non-linear, and often have fewer shocks than variables in the system, a condition referred to as stochastic singularity. As will be discussed in more detail below, several authors have suggested diagnostics for identification of DSGE models relying on parameters estimates that will be inconsistent when identification fails. Others assume that the parameters of the reduced form are well identified. This may not hold true for every reduced form representation as we see below.

We define identification in terms of the joint distribution of the data instead of the likelihood to circumvent the problem of stochastic singularity. The order condition that we will establish is based on simple counting of the number of parameters that can be estimated from the reduced form model, vis-́vis the dimension of Θ. As in classical econometric theory, the order condition is necessary but not sufficient for identification. We show how to check for 'conditional identification' in the form of the gradient of parameters that govern the autocovariance structure with respect to the parameters of interest.

Our analysis rests on the fact that the reduced form model we derive is a 'minimal representation', and as such, there is a unique mapping between the reduced form parameters and the autocovariances. The result is closely related to linear control theory which shows that if the input noise is not observed, structural identification requires a one-to-one mapping between the model parameters and the spectrum. Our approach is also related to identifiability of multivariate ARMA models using Kronecker indices in time series analysis.[1] However, because the DSGE model defines the joint distribution of the variables, we can use classical econometric arguments without direct evaluation of the spectrum or Kronecker indices. We illustrate the basic ideas using a simple one sector stochastic growth model, which will be presented in Section 2. The general reduced form is given in Section 3, while identification results are given in Sections 4 and 5. We also suggest diagnostic tools for identification strength. The method is applied to the simple stochastic growth model. Section 6 concludes.

Almost every empirical DSGE exercise only estimates a subset of the parameters and fixes many others. For example, Del Negro, Schorfheide, Smets, and Wouters (2007) fix 7 of the 47 model parameters, while Smets and Wouters (2007) fix 7 of the 39 parameters. Christiano, Motto, and Rostagno (2007) split the model parameters into two groups:- a set of 26 parameters that control the steady state which they fixed at values taken from the literature, a set of 55 parameters that control the dynamics which they estimate. Even with the simple stochastic growth model, Ruge-Murcia (2007) only estimates three of the six parameters and fixes the remaining three parameters. Our results shed light on when some of these restrictions are truly necessary. An important finding is that the dynamic specification of the shock processes has important bearing on whether the order

[1] See, for example, Solo (1986) and Lutkepohl (2005).

condition for identification holds.

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