Volatility Pricing in the Stock and Treasury Markets

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Abstract

An asset's sensitivity to stock market volatility carries a significant risk premium across both equity and fixed income markets. Large-cap and growth stocks have less exposure to volatility risk. Their relatively greater ability to weather volatility surprises, such as those often associated with financial crises, accounts for their lower expected returns when compared to small-cap and value stocks. In the Treasury market, Treasury bonds covary positively and significantly with volatility surprises, paying off in times of increased aggregate uncertainty. Therefore, Treasury market models that incorporate stock market volatility risk should provide more accurate models of expected returns for bonds. My novel three-factor model with market return, a volatility surprise factor, and the *HML* factor dominates the Fama-French three-factor model at estimating the cross-section of expected returns for the universe of assets containing both stocks and U.S. Treasuries. Further, my model also outperforms a model with predictive variables (aggregate dividend yield, term spread, default spread, and one-month Treasury-bill yield). I also document a significant and negative correlation between stock market volatility and shocks to cash flows for small-cap and value firms, which suggests that the cash flow constraints matter the most for these types of firms during bad times. This also explains why volatility subsumes the effect of a characteristic-based factor like *SMB* in asset pricing.

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Recent work emphasizes that stock market volatility is a pervasive risk factor that explains the cross-section of expected stock returns. Using data no earlier than 1963, influential papers by Ang, Hodrick, Xing, and Zhang (2006), and Adrian and Rosenberg (2008) document that a volatility surprise factor carries a negative risk premium. However, the tight link between expected stock returns and volatility risk should be evident in earlier periods and should be priced in other markets, as well.

This paper presents the following results. First, large-cap and growth stocks have less exposure to volatility risk. The portfolio theory (Merton (1973)) says that investors should expect lower returns from these securities. Second, stock market volatility has pricing implications for Treasury bonds, which supports its cross-markets impact. Third, replacing the SMB factor, in the Fama and French (1992, 1993) model, with a volatility surprise factor creates a novel three-factor model with a greater explanatory power in the cross-section of expected returns for stocks and treasuries. Fourth, this new model also outperforms a model with predictive variables (aggregate dividend yield, term spread, default spread, and one-month Treasury-bill yield). Fifth, volatility risk is tied to firm fundamentals; there is a significant negative correlation between stock market volatility surprises and shocks to cash flows for small-cap and value firms.

Stock market volatility has been related to the volatility of economic fundamentals (Schwert (1989a, 1989b), Hamilton and Lin (1996)), and has been considered a systematic source of risk (Chen (2003), Vayanos (2004)). Engle and Rangel (2005), and Engle, Ghysels, and Sohn (2006) propose models that make possible the identification of the macroeconomic forces that drive stock market volatility, like the volatility of GDP growth, inflation and short term interest rate. Recently, by also making use of information available from the options market, Bollerslev, Tauchen, and Zhou (2008), and Drechsler and Yaron (2008) show that the variance premium, defined as the difference between implied and realized variances, captures uncertainty in economic fundamentals and helps in predicting stock returns.[1]

This paper presents further evidence consistent with the role played by volatility as a systematic source of risk. The sample periods used are January 1927 to December 2007 for the stock market, and January 1952 to December 2007 for the Treasury market. They are characterized by several recessions and financial crises, that are critical for detecting volatility risk. Using such long periods also justifies volatility's role as a state variable proxying for bad time risk.

I find that volatility has a significantly negative price of risk in the stock market, which is in line with previous literature. Further, large-cap and growth stocks are less affected by volatility surprises, holding the market return fixed. Their relatively greater ability to weather volatility surprises, such as those often associated with financial crises, accounts for their lower expected returns when compared to small-cap and value stocks. A portfolio long on value stocks and short on growth stocks has an average volatility premium of 8 basis points (bp) per month, while a portfolio long on small-cap stocks and short on large-cap stocks has an average volatility premium of 41bp per month.

In the Treasury market, Treasury bonds covary positively and significantly with volatility surprises, paying off in times of increased aggregate uncertainty. This result suggests that Treasury market models that incorporate stock market volatility risk should provide more accurate models of expected returns for bonds. But more fundamental is the role played by volatility across markets, because portfolio decisions are made by allocating funds between stocks and treasuries. These assets have different risk-return profiles that make them natural hedges for each other. I find

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[1] Ang, Chen, and Xing (2006) also find a premium in the cross-section of stock returns representing compensation for downside risk, when excess market return is below its mean, which is not captured by known risk factors.

that investors require a premium for holding the risky assets (stocks), which correlate negatively to volatility surprises, but they are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate in a positive fashion (the so-called "flight-to-quality").

I also document that volatility significantly improves the fit when explaining the cross-sectional variation in expected returns across stocks and treasuries. Indeed, replacing the SMB factor, which does not have economic underpinnings, with the volatility surprise factor in the Fama-French (1992, 1993) model leads to a novel three-factor model that has a far higher R-squared than what is achievable with the Fama-French model.

Further, volatility spans most of the space of forward-looking state variables previously used for predicting returns (aggregate dividend yield, term spread, default spread, and one-month Treasury-bill yield). This result reinforces its role as an inter-temporal factor in Merton's (1973) Intertemporal Asset Pricing Model (ICAPM). My new three-factor model also outperforms, in terms of explanatory power, an asset pricing model with these predictive variables.

An important question then is what is volatility capturing besides macro-economic uncertainty, especially since I find that SMB proxies for volatility? The answer is rooted in what happens during bad times, when firms' cash flows are constrained. I document a significant and negative correlation between volatility surprises and shocks to cash flows for small-cap and value firms, which suggests that the cash flow constraints matter the most for these types of firms during volatile times, making them relatively riskier. This finding also explains why volatility subsumes the effect of a characteristic-based factor like SMB in asset pricing.

The findings here are robust. A second (unexpected) volatility factor – the difference between predicted volatility constructed from an $Asymmetric - GARCH$ model and realized volatility – provides additional evidence that volatility surprises, however computed, are important sources of risk. The results also are robust when I control for momentum and aggregate liquidity. They hold under a variety of model specifications and in different sample periods.

The economics underlying my findings lie in what happens during the most turbulent times, when market returns often are highly negative, while volatility rises in response to a series of volatility surprises. Assets with positive exposure to volatility surprises are safer during these times. To insure their portfolios against the double penalty of a declining stock market and a rise in volatility, investors are willing to pay a premium for holding them. Recent events associated with the subprime mortgage meltdown also attests to the interdependence between the stock and Treasury markets, and to their joint sensitivity to extreme market volatility events. These events led investors to swap Treasuries for stocks, that in turn led to one of the worst periods for holding stocks to coincide with one of the best periods to hold Treasuries.

The remainder of the paper is structured as follows. Section 1 derives the asset pricing model and builds unexpected volatility. Section 2 prices the risk associated with unexpected volatility. Section 3 performs robustness checks. Section 4 links volatility to predictive variables. Section 5 provides empirical evidence that volatility surprises capture shocks to firm fundamentals. Section 6 provides concluding comments. Hereafter, the term volatility refers to unexpected stock market volatility.

1 The Model

Based on Merton's (1973) model, I assume that the cross-section of assets has two sources of risk. The first source is the classic market beta, and the second is exposure to a state variable associated with volatility surprises. Formally:

$$E[R^e_{i,t+1}/\Omega_t] = \lambda_{m,t}\beta^m_{i,t} + \lambda_{UV,t}\beta^{UV}_{i,t} \qquad i = 1, 2, ..., n, \tag{1}$$

with

$$\beta^m_{i,t} = \frac{Cov\left(R^e_{i,t+1}, R^e_{m,t+1}/\Omega_t\right)}{Var\left(R_{m,t+1}/\Omega_t\right)} \quad and \quad \beta^{UV}_{i,t} = \frac{Cov\left(R^e_{i,t+1}, UV_{t+1}/\Omega_t\right)}{Var\left(UV_{t+1}/\Omega_t\right)}, \tag{2}$$

where Ω_t represents the information set available to investors at time t. The R^e_i and R^e_m represent excess returns on the asset i and the market portfolio, respectively. The UV_t represents the surprise component of volatility. Distinct from prior research, I measure volatility surprises as the residual from an AR(1) model fitted to stock market volatility. Specifically:

$$V_t = \mu_V + \phi^V_1 V_{t-1} + UV_t. \tag{3}$$

The V_t represents realized volatility and is computed as in Anderson et al. (2003), by summing up the squared daily market returns within a month, and then taking the square root of this quantity:

$$V_t = \sqrt{\sum_{i=0}^{\Delta} R^2_{m,t+i}}. \tag{4}$$

The Δ represents the number of trading days within a month. A plot of V_t for the period January 1927 to December 2007 can be seen in Figure 1. Figure 2 plots UV_t over the same time period. As expected from a proper bad time proxy, the graphs document spikes in volatility during recessions and periods of financial distress (the shaded areas in the plots).

Next, taking unconditional expectations in equation (1), the following equation obtains:

$$E[R^e_{i,t+1}] = E(\lambda_{m,t})E(\beta^m_{i,t}) + Cov(\lambda_{m,t}, \beta^m_{i,t}) + E(\lambda_{UV,t})E(\beta^{UV}_{i,t}) + Cov(\lambda_{UV,t}, \beta^{UV}_{i,t}). \tag{5}$$

Following Ang et al. (2006), the betas are assumed to be constant through time: $\beta^m_{i,t} = \beta^m_i$ and $\beta^{UV}_{i,t} = \beta^{UV}_i (\forall)\ t = 1, ..., T$.[2] Therefore, the covariance terms drop out in equation (5). Using the notation $\lambda_m = E(\lambda_{m,t})$ and $\lambda_{UV} = E(\lambda_{UV,t})$, the following equilibrium asset pricing model for the unconditional expected returns obtains:

$$E[R^e_{i,t+1}] = \lambda_m \beta^m_i + \lambda_{UV} \beta^{UV}_i \qquad i = 1, 2, ..., n. \tag{6}$$

Previous empirical studies of financial markets find evidence that the disturbance variance in the time series models is less stable than usually assumed (the heteroskedastic property). Also, autocorrelation patterns often present in the time series of returns can lead to biased standard errors for the estimates of the factors' loadings in a simple *OLS* framework. Therefore, my approach is to estimate factor loadings by explicitly incorporating the heteroskedasticity and autocorrelation of returns:

$$\begin{aligned} R^e_{i,t} &= \alpha_i + \beta^m_i R^e_{m,t} + \beta^{UV}_i UV_t + \eta_{i,t} \\ \eta_{i,t} &= \varepsilon_{i,t} - \gamma_i \eta_{i,t-1}, \\ \varepsilon_{i,t} &= \sqrt{h_{i,t}} e_{i,t}, \qquad\qquad i = 1, ..., 25. \\ h_{i,t} &= \omega_i + \phi_{1,i} h_{i,t-1} + \theta_{1,i} \varepsilon^2_{i,t-1}, \\ e_{i,t} &\sim N(0, 1). \end{aligned} \tag{7}$$

I estimate the parameters in equation (7) using the maximum likelihood method.[3]

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[2] My conclusion does not change when betas are allowed to change through time. Results are available upon request.

[3] My conclusion does not change when I use the OLS estimation technique for a simple regression model, instead of the maximum likelihood method with the specification in equation (7). Results are available upon request.

2 Pricing Volatility

2.1 *Stock Market*

The two testable implications for the model specified in Section 1 are as follows. First, there is a pattern of differential volatility loadings across the test assets, matching the pattern in their average returns for my long historical sample. Second, volatility risk is related to the size and value anomalies in the cross-section of stock returns.[4]

The time period under consideration, January 1927 to December 2007, is characterized by recessions in 1933, 1945, 1949, 1954, 1958, 1961, 1981-1982, 1983, 1991, 2000-2001. Also, major crises occurred like the Penn Central commercial paper debacle of May 1970, the oil crisis of November 1973, the stock market crash of October 1987, the Asian crisis of 1997, the Russian debt default of 1998, the burst of the hi-tech bubble of 2000, the September attack of 2001, the accounting scandals of 2002 (Enron, WorldCom), and the financial crisis of 2007 (the bursting of the US housing bubble, accompanied by high default rates on subprime and other adjustable rate mortgages) (see the shaded areas in Figure 1).

The test assets are the 25 size- and value-sorted portfolios of Fama and French (1992, 1993). Value-weighted returns for these portfolios are obtained from Kenneth French's Web site at Dartmouth (they are reported in Table I). I use monthly data in my tests, because relations in monthly returns have more economic significance and are less likely to be driven by frictions or data measurement issues, especially for small-firm returns.

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[4] See for example Banz (1981) and Reinganum (1981) for the size effect, Graham and Dodd (1934), Basu (1977, 1983), Ball (1978), Rosenberg, Reid, and Lanstein (1985) for the value effect, and Fama and French (1992, 2008) for both effects, among others.

The first step in the asset pricing exercise is to estimate the factor loadings, β_i, in equation (7). Maximum likelihood estimates with robust standard errors are reported in Table II. With a few exceptions, such as the large-cap stocks, volatility loadings are negative and statistically meaningful across the portfolios of stocks. This result was expected, because stocks experience low returns during turbulent times.[5] A more interesting result is that volatility loadings tend to (but do not always) increase with size, and are significantly negative for small-cap stocks. As an example, the smallest value stocks have a volatility beta of -0.41, while the largest value stocks have a beta of 0.12. Volatility loadings are also inversely related to the book-to-market ratio. For instance, the smallest growth stocks have a beta of -0.24, while the smallest value stocks have a volatility beta of -0.41. These patterns translate into a significant spread in risk between small-cap and large-cap stocks, as well as between value and growth stocks (with the exception being the largest stocks). Figure 3 documents the alignment between the average portfolio excess returns (Panel A) and the volatility loadings (Panel B).

Table II also supports an $IGARCH$ structure for the error variance, since the sum $(\widehat{\phi}_1 + \widehat{\theta}_1)$ is close to one for most of the portfolios (see Appendix B for a definition of $IGARCH$ models). Further, the securities most affected by volatility risk (small-cap and value stocks), exhibit a strong autoregressive pattern in their return series. The heteroskedasticity tests Portmanteau Q and Lagrange Multiplier LM strongly reject the null hypothesis of homoskedasticity in the error variance (results not reported). Also, the time series plots of the conditional volatilities for the 25 portfolio returns suggest that returns are heteroskedastic and that small-cap stocks have the highest average conditional volatility (see Figure 4). As in Bollerslev (1987), the normality test indicates that the conditional normal distribution does not fully explain the leptokurtosis present in the portfolio

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[5] This finding is also in Adrian and Rosenberg (2008) and Moise (2002).

returns (results not reported). Further, equation (7) explains at least 51% of the variation in the time series of portfolio returns. Therefore, even if the temporal dependence in returns is present only among small-cap and value stocks, heteroskedasticity poses a big problem for all stocks, supporting the specification in equation (7).

The above findings motivate me to formally test volatility effects in the cross-section of stock returns. To investigate whether volatility risk is priced in equilibrium, I estimate factor risk prices using Fama-MacBeth (1973) regressions of returns on factor betas. The statistical distribution of the estimated prices of risk is used to assess whether these factors are priced in the cross-section of stock returns. Although my results mitigate the "errors-in-variables in the regressors" problem by using returns from the broad-based Fama-French portfolio as units of observation, I still use the Shanken (1992) correction to address concerns arising from this issue.

Table III Panel A) reports the estimates for equation (6). Although there is on average a significant trade-off between market risk and return over the time frame January 1927 to December 2007, a non-beta measure of risk like volatility plays an important and apparently systematic role, with a noticeable influence on stocks' equilibrium expected returns. Controlling for the market risk, volatility has a precisely estimated price of risk of -110bp per month (t-stat = -2.96). Table III Panel B) presents volatility risk premiums (computed as the product between volatility loadings and its price of risk) across the test assets. Small-cap and the majority of value stocks entail considerably larger volatility premiums compared to large-cap and growth stocks. These larger premiums result because small-cap and value stocks are affected to a higher degree by volatility risk. For instance, volatility premiums average 33bp per month across small-cap firms versus -8bp per month across large-cap firms, and they average 20bp per month across value firms versus 12bp per month across growth firms. A portfolio long on value stocks and short on growth stocks has an average volatility premium of 8bp per month, while a portfolio long on small-cap stocks and short on large-cap stocks has an average volatility premium of 41bp per month. Controlling for market risk, the size spread in the volatility premium ranges between 29bp and 59bp per month across the book-to-market sorted groups, while the value spread ranges between -0.11bp and 20bp per month across the size-sorted groups. By comparing these spreads with the corresponding spreads in returns from Table I, I infer that the size and value effects are to a considerable extent entailed by volatility risk exposure, with stronger results in the size dimension.

My findings suggest that investors demand large-cap and growth stocks during turbulent times, since small-cap and value stocks expose them to larger downside risk. This finding implies that in the $ICAPM$ world the market portfolio is not mean-variance efficient with respect to the universe of common stocks, and suggests adding a position that takes into account stock market volatility. Since investors are not fully insured against systematic volatility risk, the size- and value-related premiums reflect their attempts to reduce this risk exposure. From an economic perspective, as a recession deepens, credit markets get tighter and profitability decreases. As a result, investors shift their preferences towards better collateralized, larger firms and towards more profitable, growth firms. In accordance to rational pricing, this hedging demand drives up the price for such stocks, and leads to lower expected returns.

2.1.1 *Controlling for other Risk Factors*

Fama and French (1996) rely on their three-factor model with market return, SMB and HML for explaining stock return anomalies related to firm characteristics.[6] Another variable proposed as a

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[6] HML and SMB are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). HML is the difference between high book-to-market-stocks portfolios and low book-to-market-stocks portfolios, with similar weighted-average size, while SMB is the difference between the returns on small-stocks portfolios and those of big-

candidate for a state variable within the ICAPM is the momentum factor (MOM) of Jegadeesh and Titman (1993).[7] To quantify the volatility effect over and above the effects of these factors, the asset pricing model in equation (6) extends to:

$$E[R^e_{i,t+1}] = \lambda_m \beta^m_i + \sum_s \lambda_s \beta^s_i, \tag{8}$$

where λ_s and β^s_i represent the price of risk, and the loading associated with the generic factor s.

Table IV Panel A) documents the average returns only for the traded risk factors, and Panel B) presents the cross-correlation matrix for these factors with volatility. There is a significant negative correlation between volatility and market return (equal to –33%), which was expected since rises in volatility correspond to downturns in the market. There is also a significant correlation between volatility and size (equal to - 18%), and between volatility and momentum (equal to -11%).

Table V presents estimates of equation (8) for various values of s, in which the betas represent factors' loadings estimated in the corresponding time series models. The R^2_{adj} statistic from the cross-sectional regression of average excess returns on the risk factors is used for comparing the relative performance of the different asset pricing models (Jagannathan and Wang (1996)). Results show a significant increase in the model's explanatory power when CAPM is augmented with volatility; CAPM has an R^2_{adj} of 1.07%, and adding volatility raises the R^2_{adj} to 51.88%. Volatility also adds to the explanatory power of the Fama-French (1992, 1993) model, by increasing its R^2_{adj} from 77.43% to 86.94%. Further, volatility subsumes the SMB effect in explaining the cross-sectional variation in stock returns. In light of these results, I replace the SMB factor in the Fama-French model with volatility, which leads to a novel three-factor model.

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stocks portfolios, with similar weighted-average book-to-market equity.

[7] MOM is built as the average return on the two high prior (months 2-12) return portfolios minus the average return on the two low prior return portfolios.

A stringent test consists of assessing the empirical performance of my new model versus the Fama-French three-factor model. I find that my model dominates the Fama-French model at explaining the cross-sectional variation of expected returns. When using my model, the R^2_{adj} increases from 77.43% to 82.17%, and the intercept drops from 1.51 to 1.23. It is also helpful to visually compare the performance of my model with the other asset pricing models. To this end, I plot for each portfolio the fitted expected return versus the realized expected return. A good fit of a model translates into the points plotting along a 45-degree line going through the origin. Figure 5 Panel A) documents the empirical failure of $CAPM$, while Panel B) shows the performance of a two-factor model with market return and volatility. Panels C) and D) present the Fama-French model and my three-factor model. The plots show that my model goes a long way in explaining the cross-sectional variation in expected stocks returns, with the smallest pricing errors.

My tests also show that volatility is robust with respect to the inclusion of the MOM factors in the model. Its price of risk ranges from -86bp to -112bp per month across the different specifications in equation (8).[8] Further, although the market, SMB and MOM risk prices are mostly imprecisely estimated, the volatility and HML risk prices are always precisely estimated. All these results indicate that both volatility and HML are significant determinants of average excess returns.

2.1.2 *Different Model Specifications*

In this section I use a different specification for the cross-sectional asset pricing model, that involves constraining the constant to be equal to zero. This specification is motivated by a comparison of Tables V and VI. Table V documents a negative and sometimes imprecise price of risk for the

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[8] Market volatility is not a traded factor. Nonetheless, one can project volatility on the space of returns to obtain a traded factor, as in Moise (2002) or Ang et al. (2006).

market return (range of -33bp to -110bp per month), that in turn implies a negative market risk premium. This result may be due to a measurement error in equation (8): the betas are generated regressors and not "true" betas (the "errors-in-variables" problem). They are noisy and could lead to a significant intercept in the cross-sectional regression. Another explanation for this result is that the market beta acts like a constant in the cross-sectional tests, since it does not vary much across the test assets. Freeing up the constant in the cross-sectional model leads to a near-perfect multi-collinearity, which translates into a high R^2 along with insignificant regressors. In order to test for multicollinearity, I choose to remove one of the "constants", the market beta, from one of the models. The result is a change in both sign and significance for the constant (see the third model in Table V), which supports my second explanation. Further, if the factors form a basis for the space of test assets, and if the factors are traded in the market, then their risk prices should be close in value to their means (Cochrane (2001)).[9] Table IV Panel A), which presents the vector of average returns only for the traded factors, and Table VI, show that this is the case when using a cross-sectional model that forces the intercept to be equal to zero.

Thus, I also test a new specification, in which the actual constant is constrained to zero in the cross-sectional test. The results have negligible implications for volatility, SMB, HML and MOM, but dramatic ones for the market factor (see Table VI). Specifically, I find that, by removing the constant from the cross-sectional asset pricing model, I obtain economically sound results. More importantly, the volatility effect is robust with respect to the new model specification, and its price of risk is still precisely estimated in the range of -63bp to -154bp per month (Table VI).

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[9] I refer only to traded factors here, thus excluding the non-traded volatility factor.

2.1.3 *Different Samples*

The model described in equation (8) is tested now using two additional samples: the post-depression sample (January 1935 to December 2007) and the post-Compustat sample (January 1963 to December 2007). To this end, I re-estimate all factors' loadings and risk prices for each sample. Also, the liquidity factor of Pastor and Stambaugh (2003) is available starting with 1963, so I include it in the model for the latter sub-period.

My results support the findings in the previous sections. Volatility risk is priced in the cross-section of stock returns in both samples (see Table VII). Volatility has a larger price of risk in the post-depression sample than in the post-Compustat sample (i.e., in the two-factor model, volatility price of risk drops from -106bp to -76bp per month). Nonetheless, it is precisely estimated in both samples, and is robust with respect to the inclusion of the other risk factors. The price of risk for the momentum factor, although imprecisely estimated in the post-depression sample, becomes significant in the post-Compustat sample, in the absence of SMB. Although the *HML* price of risk is always precisely estimated, the *SMB* factor is not priced in either sample. As before, my three-factor model dominates the explanatory power of the Fama-French model in the cross-section of expected stock returns for both samples. Further, volatility is precisely estimated even in the presence of liquidity. As a side note, the two-factor model with market return and volatility outperforms the Fama-French three-factor model in the post-Compustat sample (R^2_{adj} of 78.03% versus 74.75%; Table VII Panel B). Because this result does not hold over the longer time period, I consider it as being sample-specific, and conclude that the HML factor needs to be included in the asset pricing model in order to obtain better explanatory power.

2.2 *Treasury Market*

If stock market volatility proxies for the common underlying sources of macroeconomic risk, then it should be priced in other markets as well. In addition, if the test assets have a factor structure (as is the case with the 25 Fama-French size- and value-sorted portfolios used in the stock market tests), and if the variables tested have some correlation with these factors (as is the case with volatility and SMB), then one obtains a good model fit (see Lewellen, Nagel and Shanken (2006)). Therefore, I extend my test assets by including portfolios from the Treasury market. I use the Fama bond portfolios' returns downloaded from CRSP. Data cover the period January 1952 to December 2007, and consist of 12 portfolios with a maturity interval of six months. The first portfolio has return maturities from one to six months, and the last has return maturities greater than one hundred and twenty months. Only non-callable, non-flower notes and bonds are included in these portfolios. Portfolio returns are computed as an equal-weighted average of the unadjusted holding period returns of individual bonds.

According to the liquidity preference hypothesis, bond returns should increase with maturity, because long-term bonds are more sensitive to interest rate risk. This monotonic pattern can be observed in my sample, with average excess returns (in excess of the one-month T-bill rate) increasing with maturity, from 4bp to 19bp per month, all statistically significant (see Table VIII, Panel A).

First, I analyze the Treasury market in isolation. I estimate factors' loadings on bond portfolio returns using a two-factor model (CAPM and volatility). The volatility loadings are all positive and precisely estimated (Table VIII, Panel B), which translates into treasuries paying off in times of increased economic uncertainty, when volatility rises. Also, I find an increasing (and roughly monotonic) pattern in volatility loadings across the different maturity bonds, with long-maturity bonds having larger exposure to volatility risk. This result propels me to investigate the explanatory power of stock market volatility in the cross-section of treasuries. As in Section 3.1.2, I report the results both with and without an intercept in the cross-sectional model. My findings support the role of volatility as a risk factor priced outside the stock market. Specifically, controlling for market risk, volatility has a significant price of risk equal to 149bp per month. Its monthly premium varies from 0bp per month, for short-maturity bonds, to 28bp per month, for long-maturity bond (see Table VIII, Panels C) and D). I infer that during economic contractions investors require a premium for holding long-maturity bonds, because they have a higher exposure to volatility risk. Presumably, a forward-looking stock market variable such as volatility captures expectations about future interest rates, which explains the larger volatility premium found in long-maturity treasuries. As a side note, the market factor has a negative price of risk of -202bp per month, which translates into negative market premiums across treasuries. This result was expected, since treasuries' payoffs are lower during good times, when investors flock into the stock market. In sum, the evidence presented in this section points towards volatility exerting an effect in the Treasury market, as well. Therefore, models in the Treasury market incorporating stock market volatility are likely to provide more accurate models for expected bond returns.

2.3 *Joint Markets*

In this section I analyze the stock and Treasury markets together. Understanding the role played by volatility in the joint markets is fundamental, because portfolio decisions are made by allocating funds between stocks and bonds. More importantly, the difference in the risk-return profiles for stocks and bonds makes them natural hedges for each other.

I re-run my tests on 37 portfolios (the 25 size- and value-sorted portfolios plus the 12 maturity-sorted bond portfolios). The cross-sectional tests show that volatility has pricing implications across markets. Controlling for market risk, volatility has a significant price of risk of -102bp per month (see Table IX). As before, the volatility effect is robust with respect to the inclusion of the other risk factors in the model. Its risk price ranges from -72bp to -119bp per month, and is always precisely estimated. The (mostly) negative loadings of volatility on stocks returns (documented in Table II) and the positive loadings on bond returns (documented in Table VIII, Panel B) imply that the volatility premium is positive in the stock market and negative in the bond market. The economic motivation behind this result is that in bad times investors require a premium for holding the risky assets (stocks), which correlate negatively with volatility, and they are willing to pay a premium for holding the safe assets (Treasury bonds), which correlate in a positive fashion (the so-called "flight-to-quality").

Turning to my three-factor model, I find that it still dominates the Fama-French model in terms of the factors' significance and intercept value. Also, once the universe of test assets is expanded by including the bond portfolios, the market price of risk becomes positive, although it is still imprecisely estimated when the constant is freed up in the model. Because market betas vary between stock and bond portfolios, the near-perfect multicollinearity problem reported for the stock market is weakened, which in turn leads to an economically reasonable, although still imprecise, estimate for the price of market risk.

3 Robustness Checks - An Alternative Volatility Measure

This section ensures that results are robust with respect to the volatility measure employed. For this purpose, a second (unexpected) volatility factor is built, that is the difference between predicted and realized market volatility. To predict volatility, an underlying model that captures the following stylized facts is needed; volatility increases after a drop in stock prices (Black (1976), French, Schwert, and Stambaugh (1987), Nelson (1990)), is persistent (Schwert (1989b), French, Schwert, and Stambaugh (1987)), and is related to macroeconomic volatility and recessions (Officer (1973), Schwert (1989a, 1990), Hamilton and Lin (1996)). Another important feature of financial data is that they usually produce non-normal residuals, which is a consequence of the leptokurtosis of returns (see also the results of Subsection 2.1). Furthermore, Black (1976), among others, pointed out an asymmetry in the stock market data, that says that negative shocks to returns drive up volatility because a decline in stock prices increases leverage, which increases the expected stock return, and then leads to higher stock return volatility. Also, a positive autocorrelation of order one is usually present in an index' return series (Campbell, Lo and MacKinley (1997)).

To address all these issues, I use a $GJR\ Asymmetric\ Student-GARCH$ model with an AR(1) specification in the mean equation in order to estimate a model for the stock market volatility:[10]

---

[10] $GARCH$ models are discretized versions of continuous time stochastic volatility models. Different models like $E-GARCH$ or $GARCH-M$ could be used for predicting volatility (see, for instance Adrian and Rosenberg, 2008). Further, a $GJR\ Asymmetric\ GARCH$ model with $Normal$ errors could also be employed. Partial surveys of the big literature on volatility models can be found in Pagan and Schwert (1990), Bollerslev, Chou and Kroner (1992), Hentschel (1995), Ghysels, Harvey and Renault (1996), Campbell, Lo and MacKinley (1997, Chap. 12) and Tsay (2001), among others.

$$
\begin{aligned}
R_{m,t} &= \mu + \rho R_{m,t-1} + \eta_t \\
\eta_t &= \varepsilon_t \sqrt{h_t}, \\
\varepsilon_t / I_{t-1} &\sim Student\ (\upsilon), \qquad t = 1, ..., T. \\
h_t &= \alpha + \phi h_{t-1} + \theta^{+} \eta_{t-1}^{2+} + \theta^{-} \eta_{t-1}^{2-}, \\
\eta_t^{2+} &= \eta_t^2 1_{\{\eta_t > 0\}},\ \eta_t^{2-} = \eta_t^2 1_{\{\eta_t < 0\}}
\end{aligned}
\tag{9}
$$

The model described by equation (9) accommodates the asymmetry in the news impact curve and the fat-tail behavior of stock returns (via the Student-t distribution), and allows for non-zero first order correlation. The degrees of freedom variable for the Student-t distribution, υ, is meant to capture the excess kurtosis present in the index return data. Further, results of Subsection 2.1 point out that the conditional $Normal$ distribution does not fully capture the leptokurtosis in return data, supporting the further refinement of the model in equation (9), by specifying the $Student - t$ distribution.

Please note that equation (9) has a different purpose than equation (7). The $AR - GARCH$ specification in equation (7) is meant to precisely estimate factors' loadings in the time series models in the presence of auto-correlation and heteroskedasticity in returns. Portfolio-level conditional volatility has not been used in the cross-sectional asset pricing tests, because this paper focuses only on pricing systematic sources of risk in an ICAPM context.[11] Meanwhile, the sole purpose of equation (9) is to obtain accurate forecasts for volatility. Although one can perform the exercise outlined in equation (9) for each test asset for which equation (7) is estimated, thus estimating idiosyncratic conditional volatilities, these volatilities would be of no use here.

In order to estimate equation (9), I follow Bauwens and Lubrano (1998) and consider a Half-

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[11] See Ang et al. (2006) for a detailed treatment of idiosyncratic volatility.

Cauchy prior for υ and flat priors on finite intervals for all the other parameters. The model is estimated using a *Gibbs sampler*, which is a very popular $Markov\ Chain\ Monte\ Carlo$ ($MCMC$) method. Since the *Gibbs sampler* requires analytical knowledge of the full conditional posterior densities, and because regression models with $GARCH$ errors do not contain this knowledge, the $Griddy - Gibbs\ sampler$ is applied to bivariate posterior densities to estimate the model (see Appendix C for a description of the $Griddy - Gibbs\ sampler$ of Ritter and Tanner (1992)). Five thousand draws are kept in the $Griddy - Gibbs\ sampler$ and the one thousand initial draws are considered the burn-in sample. The posterior estimates for the model in equation (9) are reported in Table X. Similar to the findings of Schwert (2002) for the Nasdaq and S&P composite portfolio, results show that conditional volatility is persistent over time. The extent to which a volatility shock today feeds through into the next period's volatility is equal to 0.75. The leverage hypothesis of Black (1976) is also supported in this sample (only the coefficient for the negative shocks to returns is precisely estimated).

The posterior estimates of equation (9) are then used to predict stock market volatility. Figure 1 plots this series next to the realized volatility series. The two time series track each other closely. This result is due to the $GARCH$ predicting equation being a weighted average of past squared returns, with slowly declining weights.

Next, a new (unexpected) volatility factor is built, that is the difference between predicted and realized volatility. Its correlation with the other risk factors is presented in Table IV, Panel C). Similar to the first volatility measure, it has a negative and significant correlation with the size and momentum factors (-22% and -7%, respectively).

The analysis of Section 2 is then repeated with the new volatility measure. Results show that the price of risk for the second volatility factor is always precisely estimated, ranging from -59bp to

-138bp per month (see Table XI). As in Section 2, the *MOM* and *SMB* risk prices are imprecisely estimated. My three-factor model again dominates the performance of the Fama-French model in explaining the cross-sectional variation in returns (the former model has an R^2_{adj} of 80.44%, and the latter has an R^2_{adj} of 77.43%).

A visual inspection of Figure 3, Panel C) shows an increasing pattern in volatility loadings on stock returns going from small-cap to large-cap firms, and from value to growth firms, albeit not always strictly monotonic. Nonetheless, the important result is that small-cap and value firms have more exposure to volatility risk than large-cap or growth firms. Therefore, previous findings relating volatility risk to the small-firm and value effects are robust with respect to the type of volatility measure employed. In the interest of brevity, results for the joint markets using the second volatility measure are not reported (they are similar to the results obtained when using the first volatility measure and are available upon request).

4 Horse-Race Tests with Predictive Variables

In Merton's (1973) ICAPM the risk premiums are associated with the conditional covariances between asset returns and innovations in state variables that describe the time-variation in the investment opportunity set. Using the aggregate dividend yield, the term spread, the default spread, and the one-month Treasury-bill yield as predictors of returns, the literature has shown that the Fama-French factors lose their explanatory power for the cross-section of stock returns in the presence of (some of) these state variables (Hahn and Lee (2006) and Petkova (2006)). Since my three-factor model also dominates the Fama-French three-factor model in terms of explanatory power, the next test is to check its performance by comparing it to a model with the above predictive variables. To this end I construct the following: the default spread, DEF (the difference between the yields of a long-term corporate Baa bond and a long-term corporate Aaa bond); the term spread, $TERM$ (the difference between the yields of a thirty-year and a one-year government bond); the dividend yield on the S&P500 value-weighted portfolio, DY (the sum of dividends over the last 12 months, divided by the level of the index); and the one-month T-bill yield, R_f. The data series are downloaded from CRSP for the period January 1942 to December 2007. Similar to the previous literature, I build the shocks to the predictive variables as residuals from a first-order VAR system.

In the first step, I analyze the joint distribution of shocks to $DEF, TERM, DY, R_f$ and volatility. Table XII Panel A) presents the summary statistics for the actual predictive variables, and Panel B) documents estimates from the regression of volatility on the contemporaneous shocks to predictive variables. I find that $TERM, R_f$ and DY are significantly correlated with volatility (the first two terms are positively correlated, and the third one is negatively correlated). This result implies that volatility spans the set of forward-looking state variables, and justifies its role as an inter-temporal hedging factor in the ICAPM. But the crucial question is whether volatility has an effect on the cross-section of stock returns in the presence of these state variables.

Therefore, in the second step, I analyze whether the strong correlation between these predictive variables and volatility leads to the explanatory power of volatility in the cross-section of stock returns. When volatility is added to the model with predictive variables, the model's explanatory power increases from 63.04% to 76.47%, the DEF and DY become insignificant, and the R_f drops considerably in significance and magnitude (see Table XIII). This finding implies that volatility captures most of the information contained in the predictive state variables. Further, when HML is also included in the model, all of the predictive variables become insignificant determinants of returns. This finding implies that volatility and HML span the entire set of information included in the predictive variables, which represents an improvement on the results in Hahn and Lee (2006), and Petkova (2006). An additional test consists of comparing the performance of my three-factor model with the model with predictive variables. Interestingly, my parsimonious three-factor model outperforms the model with predictive variables, achieving the highest explanatory power in the cross-section of stock returns (R^2_{adj} of 82.08% versus 76.47%).

5 Volatility and Shocks to Cash Flows

The results in the previous section support volatility's role as a risk factor proxying for changes in the investment opportunity set. An important question then is what is volatility capturing besides macro-economic uncertainty, especially since it explains the small-firm and (part of the) value premiums? The answer is rooted in what happens during volatile times, when firms' cash flow generation becomes more constrained. But does this constraint matter more for small-cap and value firms then for large-cap and growth firms?

To address this question, I focus on firm-level cash flows and their correlation with the volatility factor. I obtain quarterly data from Compustat for the period January 1962 to December 2007 and I build firm-level market equity, book equity and cash flows. Market equity is the product between the closing price and the number of common shares outstanding, book equity is the common equity, and the cash flow variable is the ratio of income before extraordinary items plus depreciation and amortization, over total assets.[12] I re-construct the volatility measure from Section 1 at a quarterly frequency and match it with the Compustat data. I then build two sets of portfolios. In the first set, I sort stocks into deciles based on market equity, and in the second one I sort them based on market-to-book equity. I group the bottom 30% of stocks sorted by market equity into a portfolio

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[12] See Deshmukh and Vogt (2005), among others, for a detailed treatment of different measures of cash flows.

of small stocks, the mid 40% into a portfolio of medium stocks, and the top 30% into a portfolio of big stocks. Similarly, I group the bottom 30% of stocks sorted by book-to-market equity into a portfolio of growth stocks, the mid 40% into a portfolio of mid-value stocks, and the top 30% into a portfolio of value stocks. For each portfolio, I then compute average cash flows. I repeat this exercise on a quarterly basis.

The dynamics in the time series of volatility and cash flows are presented in Figure 6. Cash flows to SMB (HML) are constructed as the difference in cash flows between small-cap (value) and big-cap (growth) portfolios. There is an interesting pattern in these time series; starting from early 1980's the cash flows for SMB move into negative territory, and the cash flows for HML move into positive territory. Also, the shocks to cash flows to SMB, and to a smaller extent to HML, seem to take place especially in times of increased uncertainty in the economy, as reflected in a higher volatility. Therefore, it is reasonable to test in a formal way the relation between volatility and firm-level cash flows.

To this end, I run time series tests where the regressand is the shocks to portfolio cash flows (constructed as the residual from an AR(1) model fitted to the cash flows series), and the regressor is the volatility factor. I do my analysis separately for the size and book-to-market groups. Results presented in Table XIV show a strong relation between shocks to cash flows and volatility. In the size dimension, there is an increasing pattern in volatility loadings on cash flows, when going from small-cap to large-cap stocks. For instance, volatility beta goes from -0.31 (t-stat = - 2.17) for small firms, to -0.01 (t-stat = -0.68) for medium-sized firms, and to 0.01 (t-stat=0.60) for large-cap firms. This result matches the pattern in volatility loadings on returns presented in Table II. In the book-to-market dimension, shocks to cash flows for growth and mid-value stocks don't covary with volatility, although those for value stocks do (volatility beta equals -0.04, with a t-stat of -2.90).

This result again matches the findings in Table II, where volatility loadings on returns tend to be lower for value versus growth stocks in most cases. My conjecture is that small-cap (and to a lesser extent value) firms can not invest optimally due to the constraints they face in generating their cash flows during bad times, and thus become relatively more risky.

In sum, both small-cap and value stocks experience negative shocks to their cash flows in times of heightened volatility. This finding leads to another result, that says that volatility captures shocks to firm fundamentals in the size and value dimension (with a weaker effect in the value dimension). This finding explains why volatility subsumes the effect of a characteristic-based factor like SMB in assets pricing. The fact that results are weaker in the value dimension explains why I need to keep the HML factor in my asset pricing model.

6 Conclusions

Volatility changes through time in a stochastic and fairly persistent fashion, that leads to sizeable capital gains or losses. This persistence in volatility implies that the trade-off between volatility risk and asset returns moves in a predictable way over the business cycle. Therefore, volatility plays a leading role in helping investors and policy makers better understand the market.

In this paper I link volatility risk to the size and value effects, which delivers additional insights into some of the economic fundamentals underlying these anomalies and supports the concept of market efficiency. Large-cap and growth stocks are less affected by volatility risk. Their relatively greater ability to weather volatility surprises, such as those often associated with financial crises, accounts for their lower expected returns when compared to small-cap and value stocks. This finding suggests that, when faced with a downturn in the economy, that results in tighter credit markets, investors shift their preferences towards the low risk firms, which are the better collateralized, big firms, and the more profitable, growth firms. In doing so, they are willing to forgo expected returns to get downside protection.

I also provide evidence for the cross-market effects of stock market volatility, by documenting a significant impact in the Treasury market. Therefore, models in the Treasury market that incorporate stock market volatility are likely to provide more accurate models for expected bond returns. But more fundamental is the role played by volatility across markets, because portfolio decisions are made by allocating funds between stocks and treasuries. I find that investors require a premium for holding the risky assets, which correlate negatively to volatility surprises, but they are willing to pay a premium for holding the safe assets, which correlate in a positive fashion.

I document that the volatility effect is not subsumed by any of the size, value, momentum, or aggregate liquidity effects. Also, that it is robust with respect to the type of volatility measure used, and that it is not sample specific or model dependent. Interestingly, replacing the SMB factor, which does not have economic underpinnings, with the volatility factor in the Fama-French model, leads to a novel three-factor model with superior performance.

I also find that volatility spans most of the set of forward-looking state variables (aggregate dividend yield, term spread, default spread, and one-month T-bill yield), which re-enforces its role as an inter-temporal factor in the ICAPM. Additionally, I document that my three-factor model outperforms a model with these predictive variables.

Finally, I tie volatility risk to fundamentals, by showing that volatility captures shocks to cash flows for small-cap and value firms. This result explains why volatility subsumes the effect of a characteristic-based factor like SMB in asset pricing tests.

In sum, my findings suggest that the size and value premiums have been tied to economic fundamentals for over 80 years, and the same premiums are found in the term premium of the Treasury curve for the more than 50 years over which I have data. Merton's (1973) model suggests using factors that capture unanticipated changes in the opportunity set as hedging instruments. Volatility is clearly related to such changes. By capturing the key role played by volatility in asset pricing as a hedging factor, and by extending its pricing implications outside the stock market, I have taken one step in showing that asset pricing models may work better than previously thought.

Appendix A: *GARCH* Models

Market volatility displays time series properties like clustering, autoregression and heteroskedasticity. A realized volatility estimate ignores these properties. More sophisticated statistical models are needed to capture the time-variation in returns. Simple filters, such as the rolling standard deviation used by Officer (1973), have led to parametric *ARCH* (*Autoregressive Conditionally Heteroskedastic*) or stochastic volatility models. Further, Engle (1982) introduced *ARCH* models for describing stochastic volatility, which proved to be helpful in analyzing financial time series. The *ARCH* model has, among other problems, the weaknesses of assuming symmetric effects of return shocks on volatility and a big set of parameters needed for describing volatility. A better alternative for modeling persistent movements in volatility, more parsimonious and also more flexible, has been introduced by Bollerslev in 1986 and is called *GARCH* (*or Generalized ARCH*). *GARCH* models characterize the conditional distribution of returns innovations by imposing serial dependence on their conditional variance. Large disturbances, positive or negative, become part of the information set used to construct the variance forecast for the next period's disturbance. This way large shocks are allowed to persist, to capture the volatility clustering phenomenon.

Another feature of financial time series is that negative innovations in stock returns have a greater impact on volatility than do positive ones (the leverage hypothesis). In response to this issue, the *exponential GARCH* model of Nelson (1991), the *ARCH* model of Zakoian (1994), and the *GJR model* of Glosten, Jagannathan and Runkle (1993) brought in several specifications designed to introduce asymmetry into the model. The model used in this study is the *GJR model*:

$$
\begin{aligned}
R_{m,t} &= \mu + \rho R_{m,t-1} + \eta_t \\
\eta_t &= \varepsilon_t \sqrt{h_t}, \\
\varepsilon_t / I_{t-1} &\sim \mathit{Student}\ (\upsilon), \qquad t = 1, ..., T. \\
h_t &= \alpha + \phi h_{t-1} + \theta^{+} \eta_{t-1}^{2+} + \theta^{-} \eta_{t-1}^{2-}, \\
\eta_t^{2+} &= \eta_t^2 1_{\{\eta_t > 0\}},\ \eta_t^{2-} = \eta_t^2 1_{\{\eta_t < 0\}}
\end{aligned}
\tag{A1}
$$

The distribution of $R_{m,t}$ is *Student* with mean zero and variance $h_t^{1/2}\upsilon/(\upsilon - 2)$ given past information I_{t-1} and assuming $\upsilon > 2$. The ε_t sequence is independent and the initial variance is a known constant. Let γ denote the parameter vector $(\mu, \rho, \alpha, \theta^{+}, \theta^{-}, \phi, \upsilon)$ in this model. The posterior density for a sample of T observations is given by

$$
\varphi(\gamma/R) \propto \varphi(\gamma) l(\gamma/R), \tag{A2}
$$

with the likelihood function given by

$$
l(\gamma/R) \propto \prod_{t=1}^{T} \frac{\Gamma(\frac{\upsilon+1}{2})}{\Gamma(\frac{\upsilon}{2})} (\upsilon h_t^{1/2})^{-1/2} \left[1 + \frac{R_t^2}{\upsilon h_t^{1/2}} \right]^{-\frac{\upsilon+1}{\upsilon}} \tag{10}
$$

where the prior density, $\varphi(\gamma)$, needs to respect the positivity restrictions on the parameters and the condition $\phi < 1$. Integrability of the posterior density depends in part on the integrability of the prior density. Given an integrable (or proper) prior and a non-pathological likelihood, the posterior will also be integrable. Examining the likelihood function (12) it can be seen that, if $h_t^{1/2}$ is strictly positive, since the *Student* density is finite and positive, no pathology appears. Therefore, a flat prior (like the uniform) can be used for these parameters. However, the posterior density of υ is not integrable when using a flat prior (see Bauwens and Lubrano, 1998). For the posterior density of υ to be integrable, the prior information must be such that the posterior is forced to go to zero quickly enough in the tail. The prior at the right tail should be at least $O(\upsilon^{1+d})$, with d being small and positive, e.g. $1/\upsilon^2$ (improper prior obtained by being flat on $1/\upsilon$). This prior must be truncated to the interval (m, ∞), with m being small and positive, to avoid causing problems at the left tail. This approach avoids the problem of $l(\gamma/R)/\upsilon^2$ approaching infinity as υ approaches zero. For a proper prior for υ, a $half - right\ Cauchy$ centered at 0 is used:

$$\varphi(\upsilon) \propto (1 + \upsilon^2)^{-1} \quad (\upsilon > 0). \tag{A3}$$

Some of the other possibilities for the prior on υ include a flat prior on υ over a finite range $(0, M)$, and also an exponential density (Geweke, 1993), which uses a subjective parameter chosen to fix the prior mean and variance of υ (see Bauwens and Lubrano, 1998).

Appendix B: The *IGARCH* Model

The $GARCH$ model is estimated using the maximum likelihood method. The log-likelihood function is computed from the product of all conditional densities of the prediction error:

$$l = \sum_{t=1}^{T} \frac{1}{2} \left[-\ln(2\pi) - \ln(h_t) - \frac{\varepsilon_t^2}{h_t} \right] \tag{B1}$$

The condition

$$\sum_{i=1}^{q} \theta_i + \sum_{j=1}^{p} \varphi_j < 1 \tag{B2}$$

implies that the $GARCH$ process is weakly stationary. In the presence of autocorrelation, the stationarity condition is

$$\frac{1}{1-\gamma^2} \sum_{i=1}^{q} \theta_i + \sum_{j=1}^{p} \varphi_j < 1 \tag{B3}$$

When the model is integrated in variance ($IGARCH$),

$$\sum_{i=1}^{q} \theta_i + \sum_{j=1}^{p} \varphi_j = 1 \tag{B4}$$

The interesting feature of $IGARCH$ models is that they are strongly stationary, although not weakly stationary.

Appendix C: The *Griddy-Gibbs Sampler*

The *Gibbs sampler* of Geman and Geman (1984) and Gelfand and Smith (1990) is a very popular $MCMC$ method. Let $\theta_1,\ \theta_2,\ ...\theta_n$ be a set of parameters that need to be estimated, X the available data, and M the model entertained. Suppose that the conditional distributions of each parameter given the others, $f_i(\theta_i/\theta_{j\neq i}, X, M)$ are known, but the likelihood function of the model is hard to obtain. What I do is to draw a random number from each of these conditional distributions. For instance, if $n = 3$, let's consider $\theta_{2,0}$and $\theta_{3,0}$ two arbitrary starting values of θ_2 and θ_3. Then

1. A random sample $f_1(\theta_1/\theta_{2,0}, \theta_{3,0}, X, M)$ is drawn, call it $\theta_{1,1}$;
2. A random sample $f_2(\theta_2/\theta_{3,0}, \theta_{1,1}, X, M)$ is drawn, call it $\theta_{2,1}$;
3. A random sample $f_3(\theta_3/\theta_{2,1}, \theta_{1,1}, X, M)$ is drawn, call it $\theta_{3,1}$.

This is a *Gibbs iteration.* The iteration can be repeated for n times, with n sufficiently large such that $m < n$ initial random draws can be discarded. I get the *Gibbs sample* this way, $(\theta_{1,m+1}, \theta_{2,m+1}, \theta_{3,m+1}), ...(\theta_{1,n}, \theta_{2,n}, \theta_{3,n})$,which can be used to obtain the point estimates and the variances of the three parameters.

In the case when the conditional posterior distributions of the parameters don't have closed-form expressions, the *Gibbs sampler* implementation can become complicated. But Ritter and Tanner (1992) have a method to obtain draws in this case. It is called the $Griddy-Gibbs\ sampler$:

1. A grid of points are chosen from a properly selected interval of θ_i, say $\theta_{i1} \leq \theta_{i2} \leq ... \leq \theta_{im}$. The conditional posterior density function is evaluated to obtain $w_j = f(\theta_{ij}/\theta_{lk\neq ij}, X, M)$ for $j = 1, ...m$;

2. $w_1, ..., w_m$ are used to obtain an approximation to the inverse cumulative distribution function of $f(\theta_{ij}/\theta_{lk\neq ij}, X, M)$;

3. A $Uniform(0,1)$ random variate is drawn and the observation is transformed via the approximate inverse CDF to obtain a random draw for θ_i.

The usual *Gibbs sampler* cannot be applied to the $GARCH$ model even if the error term is (conditionally) normal. It requires analytical knowledge of the full conditional posterior densities. Regression models with $GARCH$ errors do not contain this knowledge. To handle this, I apply a unidimensional deterministic integration rule to each coordinate of the posterior density in combination with the *Gibbs sampler*, as described by Bauwens and Lubrano (1998). The random draws of the joint posterior are then obtained by evaluating and inverting the full conditional densities.

I estimate the model in equation (9) using a $Griddy-Gibbs\ sampling$ algorithm, in which 5000 draws are kept and 1000 initial draws are the burn-ins sample. The grid I do the search over is similar to the one used by Bauwens and Lubrano (1998):

$\mu \times \rho \times \alpha \times \phi \times \theta^+ \times \theta^- \times v \in (-0.60, 0.94) \times (0.00, 0.40) \times (0.01, 0.90) \times (0.35, 0.95) \times (0.00001, 0.50) \times (0.01, 0.70) \times (0.01, 30)$.

References

[1] Adrian, T. and J. Rosenberg, 2009, Stock returns and volatility: pricing the short-run and long-run components of market risk, Forthcoming, *Journal of Finance* 63, 2997-3030.

Anderson, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 2003, Modeling and forecasting realized volatility, *Econometrica*.

Ang, A., J. Chen and Y. Xing, 2006, Downside risk, *Review of Financial Studies*, 19, 1191-1239.

Ang, A., R.J. Hodrick, Y. Xing, Y. and X. Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance*.

Ball, R., 1978, Anomalies in relationships between securities' yields and yield-surrogates, *Journal of Financial Economics* 6, 103—126.

Banz, R. W., 1981, The relation between return and market value of common stocks, *Journal of Financial Economics* 9, 3—18.

Basu, S., 1977, Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis, *Journal of Finance* 32, 663—682.

Basu, S., 1983, The relationship between earnings yield, market value, and return for NYSE common stocks: further evidence, *Journal of Financial Economics* 12, 129—156.

Bauwens, L. and M. Lubrano,1998, Bayesian inference on GARCH models using the Gibbs sampler, *Econometrics Journal* 1, C23-C46.

Black, F., 1976, Studies of stock price volatility changes, *Proc. Am. Stat. Assoc., Bus. Econ. Stat. Section*, 177-81.

Bollerslev, T., 1986, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307-27.

Bollerslev, T., 1987, A conditionally heteroskedastic time series model for speculative prices and rates of return, *The Review of Economics and Statistics*, 69, 542-547.

Bollerslev, T., R.Y. Chou and K.F. Kroner, 1992, ARCH modeling in finance, *Journal of Econometrics* 52,5-59.

Bollerslev, T., G, Tauchen and H. Zhao, 2008, Expected stock returns and variance risk premia, ERID Working Paper No. 5.

Campbell, J.Y., W. Lo and A.C. MacKinley, 1997, *The econometrics of financial markets* (Princeton: Princeton University Press).

Chen, J., 2003, Intertemporal CAPM and the cross-section of stock returns, Unpublished manuscript, University of Southern California.

Cochrane, J., 2001, *Asset pricing* (Princeton University Press).

Deshmukh, S., and S.C. Vogt, 2005, Investment, cash flow, and corporate hedging, *Journal of Corporate Finance*, 628-644.

Drechsler, I. and A. Yaron, 2008, What's vol got to do with it, Unpublished manuscript, University of Pennsylvania.

Engle, R.F., 1982, Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* 50, 987-1007.

Engle, R.F., E. Ghysels, and B. Sohn, 2006, On the economic sources of stock market volatility, Unpublished manuscript, NYU and UNC.

Engle, R., and J. Rangel, 2005, The spline Garch model for unconditional volatility and its global macroeconomic causes, Unpublished manuscript, NYU and UCSD.

Fama, E. F. and K.R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, E. F. and K.R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, E. F. and K.R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 60, No. 1, 396-425.

Fama, E. F. and K.R. French, 2008, The value premium and the CAPM, *Journal of Finance* 61, No. 5, 2163-2185.

Fama, E. F. and J. MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 71, 607-636.

French, K.R., G.W. Schwert, G.W. and R.F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics*, 19, 3-30.

Gelfand, A.E. and A.F.M. Smith, 1990, Sampling based approaches to calculating marginal densities, *Journal American Statistical Association* 85, 398-409.

Geman, S. and D. Geman, 1984, Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images, *IEEE Trans. Pattern Anal. Mach. Intell* 6, 721-741.

Geweke, J., 1993, Bayesian treatment of the independent Student-t linear model, *Journal of Applied Econometrics* 8 (Supplement), 19-40.

Ghysels, E., A. Harvey, and E. Renault, 1996, *Stochastic volatility, Handbook of Statistics, Vol. 14* (G.S.Maddala, ed., North-Holland, Amsterdam).

Glosten, L. R., R. Jagannathan and D.E. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance* 48, 1779-801.

Graham, B. and D.L. Dodd, 1934, *Security analysis* (McGraw Hill, New York).

Hamilton. J. D. and G. Lin, 1996, Stock market volatility and the business cycle, *Journal of Applied Econometrics*, Vol. 11, No. 5, 573-593.

Hanh, J., and H. Lee, 2006, Yield factors as alternative risk factors for size and book-to-market, *Journal of Financial and Quantitative Analysis.*

Hentschel, L., 1995, All in the family: nesting symmetric and asymmetric GARCH models, *Journal of Financial Economics*, 39, 71-104.

Jagannathan, R. and Z. Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3-53.

Jegadeesh, N. and S. Titman, 1993, Returns to buying winners and selling losers: implications for the stock market efficiency, *Journal of Finance* 48, No. 1, 65-92.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2006, A skeptical appraisal of asset-pricing tests, Unpublished manuscript, Dartmouth, Stanford, Emory and NBER.

Merton, R.C., 1973, An intertemporal asset pricing model, *Econometrica*, 41, 867-887.

Moise, C.E., 2002, Is market volatility priced? Unpublished manuscript, Case Western Reserve University.

Nelson, D.B., 1990, Stationarity and persistence in GARCH (1, 1) model, *Econometric Theory* 6, 318-334.

Nelson, D.B., 1991, Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, 59, 347-370.

Officer, R.R., 1973, The variability of the market factor of New York Stock Exchange, *Journal of Business* 46, 434-453.

Pagan, A.R., and G.W. Schwert, 1990, Alternative models for conditional stock volatility, *Journal of Econometrics* 45, 267-290.

Pastor, L., and R. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Petkova, R., 2006, Do the Fama-French factors proxy for innovations in predictive variables? *Journal of Finance* 61, 581 - 612.

Reinganum, M. R., 1981, Misspecification of capital asset pricing: empirical anomalies based on yields and market values, *Journal of Financial Economics* 9, 19—46.

Rosenberg, B., K. Reid, and R. Lanstein, 1985, Persuasive evidence of market inefficiency, *Journal of Portfolio Management* 11, 9—17.

Ritter, C. and M.A. Tanner, 1992, Facilitating the Gibbs sampler: the Gibbs stopper and the Griddy-Gibbs sampler, *Journal of American Statistical Association* 87, 861-8.

Schwert, G.W., 1989a, Business cycles, financial crises and stock volatility, Carnegie-Rochester Conference Series on Public Policy 31.

Schwert, G.W., 1989b, Why does stock market volatility change over time? *Journal of Finance* 44, No 5, 1115-1153.

Schwert, G.W., 1990, Stock market volatility, *Financial Analysts Journal*, May-June 1990.

Schwert, G.W., 2002, Stock volatility in the new millennium: how wacky is NASDAQ? *Journal of Monetary Economics* 49, 3-26.

Shanken, J., 1992, On the estimation of beta pricing models, *Review of Financial Studies* 5, 1-34.

Tsay, R., 2001, *Analysis of financial time series* (John Wiley and Sons, Inc.).

Zakoian, J-M., 1994, Threshold heteroskedastic models, *Journal Econ. Dyn. Control* 18, 931-55.

Vayanos, D., 2004, Flight to quality, flight to liquidity, and the pricing of risk, NBER Working paper.

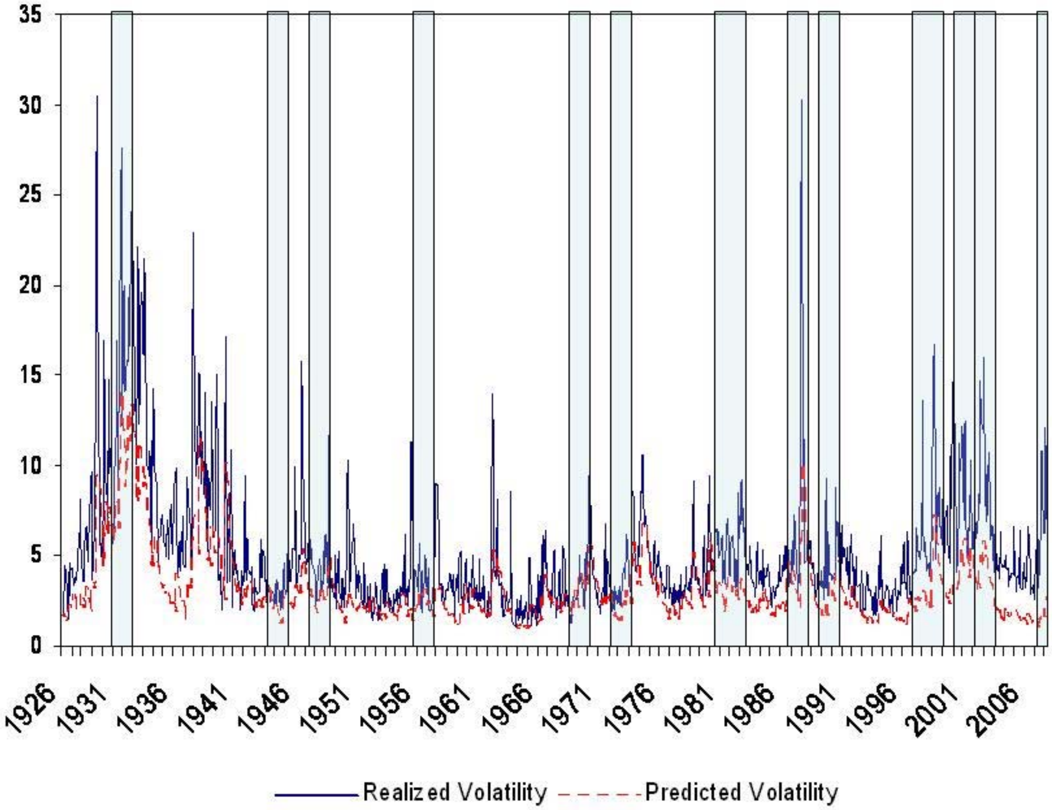


Figure 1. Realized and Predicted Stock Market Volatilities. Realized volatility is built on a monthly basis for the period January 1927 – December 2007, using daily return data from NYSE, AMEX, and NASDAQ maintained by CRSP. It is given by:

$$\nu_t = \sqrt{\sum_{i=0}^{\Delta} R_{m,t+i}^2}$$

where $R_{m,t}$ denotes daily return on the stock market portfolio and Δ represents the number of trading days in a given month. Predicted volatility is computed based on the estimates of an *Asymmetric-Student GARCH (1,1)* model applied to monthly stock market return data (see equation (9) in the text). Both series are reported in percentages, at the monthly level. Realized volatility is slightly larger than the predicted volatility. For each year, the tick marks correspond to the month of January. The shaded areas represent NBER recessions or financial distress periods.

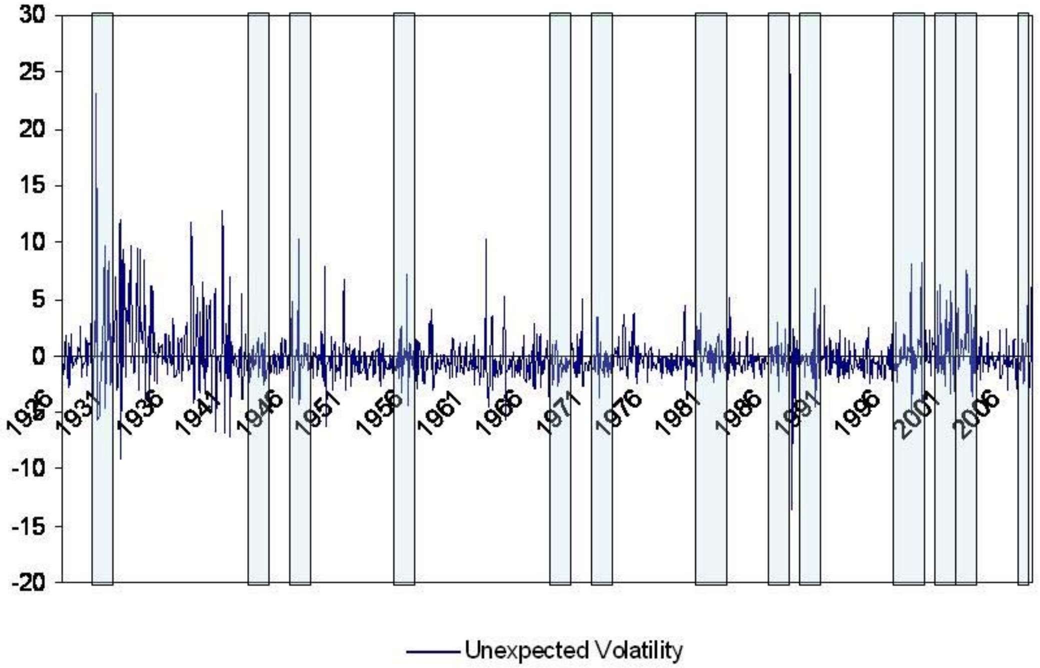
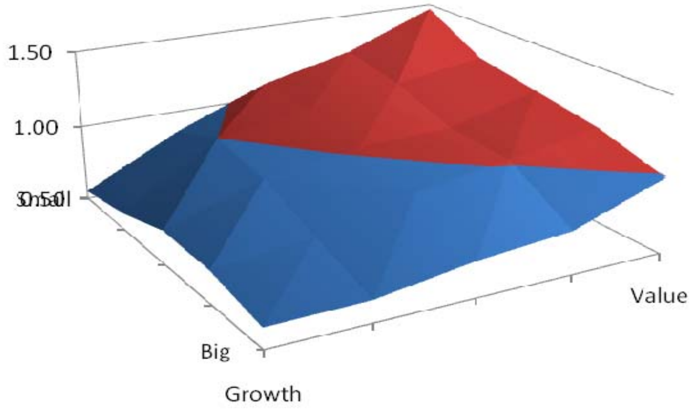
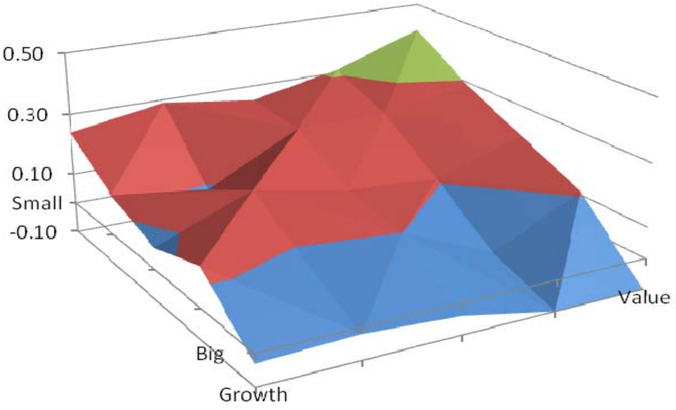


Figure 2. Unexpected Stock Market Volatility. Unexpected volatility is the residual from an AR(1) model applied to the time series of realized stock market volatility for the period January 1927 to December 2007. This series is reported in percentages at the monthly level. For each year, the tick marks correspond to the month of January. The shaded areas represent NBER recessions or financial distress periods.

Panel a) Average Excess Returns



Panel b) (–Volatility) Loadings $\hat{\beta}_{\mathrm{i}}^{-\mathrm{IV}}$



Panel c) (–Volatility) Loadings $\hat{\beta}_{\mathrm{i}}^{-\mathrm{IV}}$ *for the Second Volatility Measure*

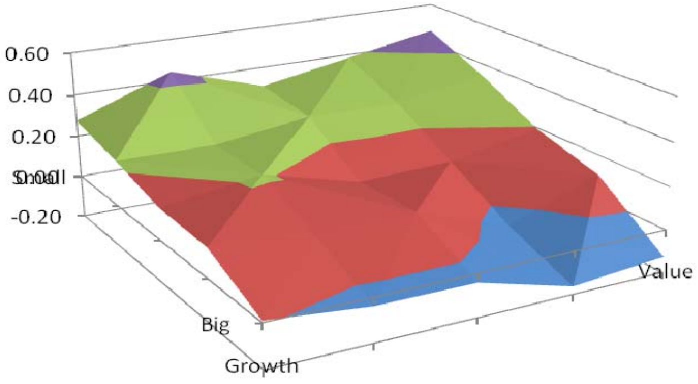


Figure 3. Average Excess Returns and Volatility Loadings for Portfolios of Stocks. Monthly value-weighted portfolio return data collected for the period January 1927 to December 2007 for the 25 Fama-French (1992) portfolios sorted on size and book-to-market equity are obtained from Kenneth French's Web site at Dartmouth. Factors' loadings are estimated using equation (14) in the text for both measures of volatility. Since most volatility loadings are negative, the figures present –loadings for better visualization.

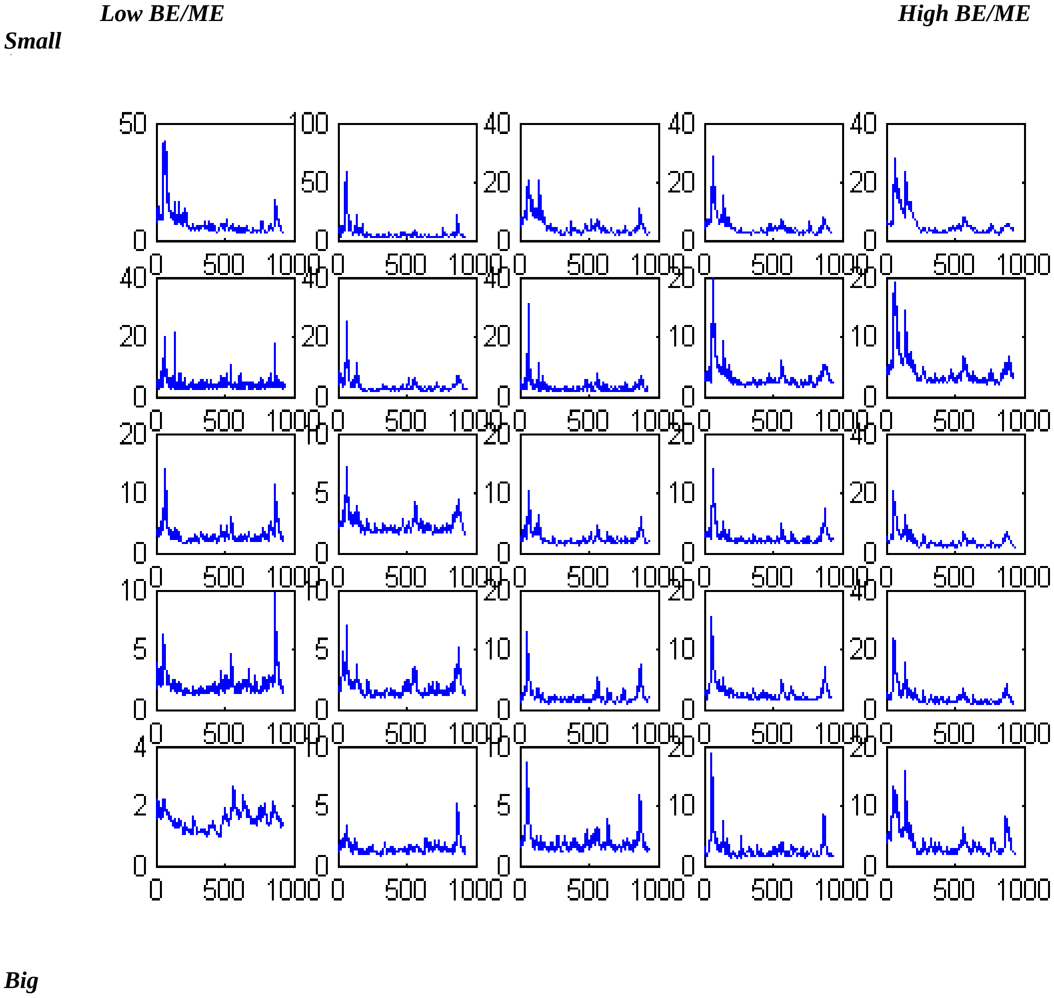
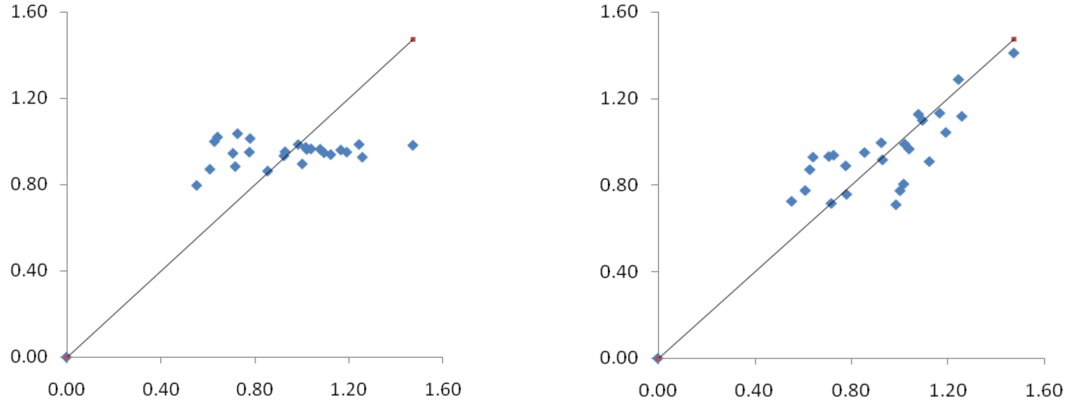
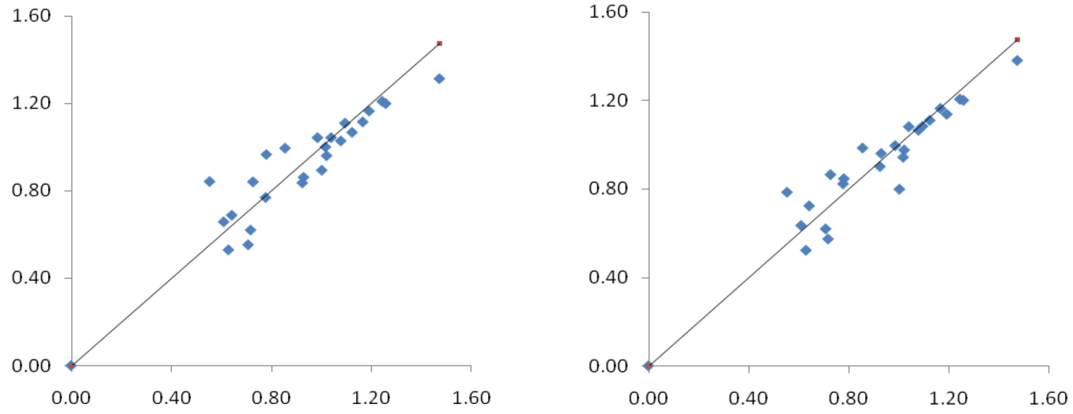


Figure 4. Conditional Volatilities. Conditional volatilities are built for the period January 1927 to December 2007 for the 25 Fama-French (1992) value-weighted portfolios sorted on size and book-to-market equity, using equation (13) in the text. The rows above index firm size, and the columns index firm value. Conditional volatilities are reported in percentages.



Panel a) The CAPM

Panel b) The Two-Factor Model (XMKT, UV)



Panel c) The Fama-French three-Factor Model

Panel d) My three-Factor Model (XMKT, UV, HML)

Figure 5. Predicted Versus Realized Average Excess Returns. Predicted excess returns are plotted on the vertical axis, and realized excess returns are plotted on the horizontal axis, using four different asset pricing models: the CAPM, the two-factor model (consisting of excess market return and its volatility), the Fama-French (1992) three- factor model, and the three-factor model proposed in this paper (consisting of excess market return, its volatility and HML). Data represent the value-weighted portfolios sorted on size and book-to-market equity and cover the period January 1927 to December 2007.

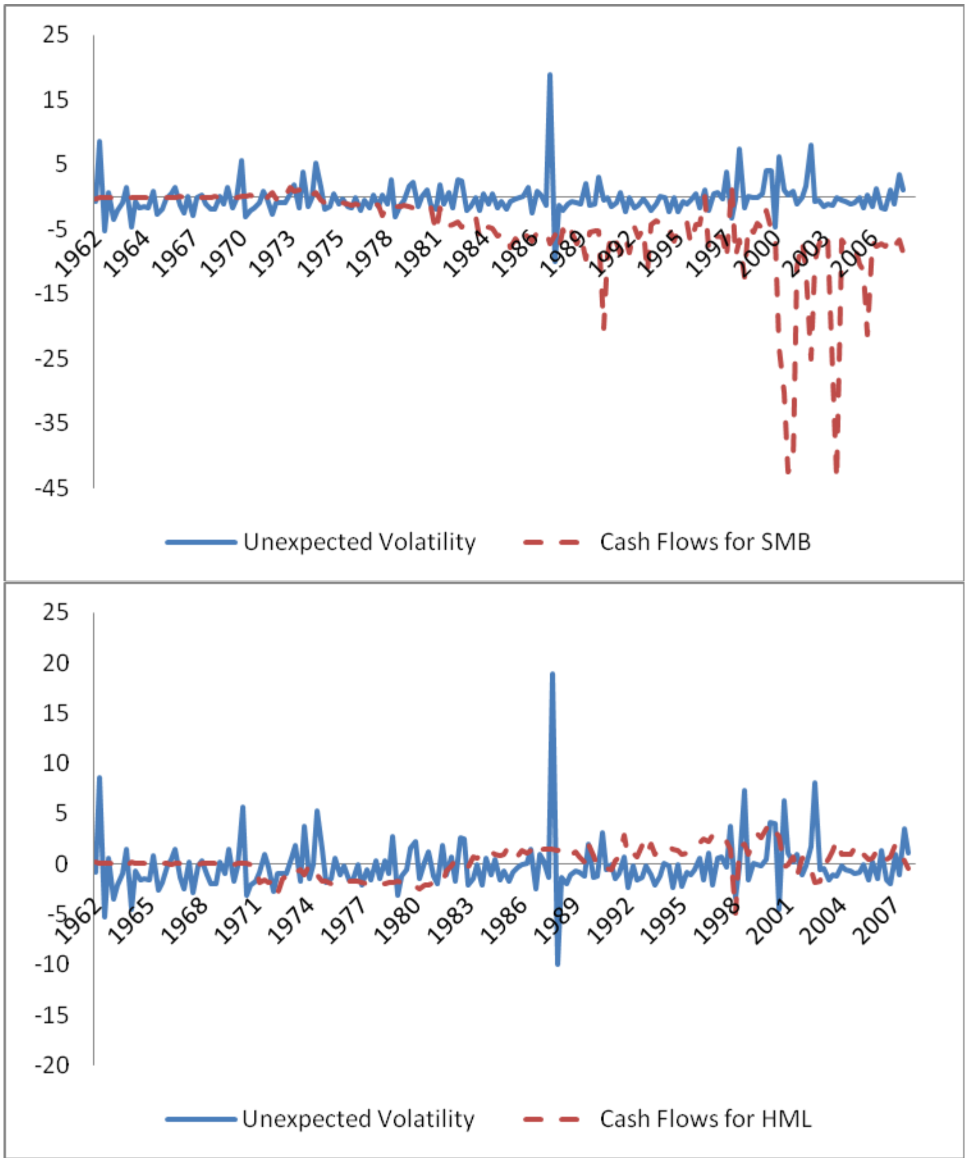


Figure 6. Unexpected Stock Market Volatility and Cash Flows. Unexpected volatility is the residual from an AR(1) model applied to the time series of quarterly realized stock market volatility for the period January 1962 to December 2007. This series is reported in percentages. The cash flows to SMB factor is the difference in equal-weighted average cash flows between two portfolios of stocks: one formed with the bottom 30% and the other formed with the top 30% of stocks sorted by market equity. The cash flow to HML factor is built in a similar manner by using stocks sorted by book-to-market equity (see text for details about the construction of the individual variables). For each year, the tick marks correspond to the first quarter.

Table I
Average Excess Returns for the 25 Fama-French Portfolios

Monthly value-weighted average excess returns for the 25 size- and value-sorted portfolios of Fama and French (1992) are reported in percentages. Data cover the period January 1927 to December 2007. The *t*-statistics are documented in parentheses.

	Book-to-Market Equity (BE/ME) Quintiles											
Size Quintiles	*Low*	*2*	*3*	*4*	*High*	*High-Low*	Low	*2*	*3*	*4*	*High*	*High-Low*
	Average Excess Returns						*t-stats*					
Small	0.55	0.86	1.10	1.24	1.47	0.92	(1.40)	(2.47)	(3.78)	(4.46)	(4.79)	(3.69)
2	0.61	1.00	1.08	1.17	1.26	0.65	(2.36)	(3.94)	(4.57)	(4.78)	(4.51)	(4.03)
3	0.72	0.93	1.02	1.04	1.19	0.47	(2.91)	(4.42)	(4.71)	(4.80)	(4.32)	(3.05)
4	0.71	0.78	0.93	1.02	1.12	0.42	(3.54)	(3.87)	(4.60)	(4.56)	(3.93)	(2.17)
Big	0.63	0.64	0.73	0.78	0.99	0.36	(3.59)	(3.83)	(3.98)	(3.55)	(4.03)	(2.10)
Small-Big	-0.08	0.21	0.37	0.46	0.49		-(0.25)	(0.70)	(1.80)	(2.62)	(2.53)	

Table II
Estimates of an Unconditional ICAPM

Monthly value-weighted return data collected for the period January 1927 to December 2007 for the 25 Fama-French (1992) portfolios sorted on size and book-to-market equity are obtained from Kenneth French's Web site at Dartmouth. The time series of portfolios' excess returns *($R^e_{i,t}$)* are regressed on the excess market return *($R^e_{m,t}$)* and UV_t in a model in which the error follows an *AR(1)-IGARCH(1,1)* specification:

$$\begin{cases} R^e_{i,t} = \alpha_i + \beta_i^m R^e_{m,t} + \beta_i^{UV} UV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} & i = 1,\ldots,25; t = 1,\ldots,T \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim \mathrm{N}(0,1) \end{cases}$$

UV_t represents unexpected volatility and is the residual from an AR(1) model applied to the time series of realized stock market volatility. The estimation is done using the maximum likelihood method. The *t*-statistics are reported in parentheses.

			Book-to-Market Equity (BE/ME) Quintiles							
Size Quintiles	*Low*	*2*	*3*	*4*	*High*	*Low*	*2*	*3*	*4*	*High*
			$\hat{\alpha}_i$					*t-stats for* $\hat{\alpha}_i$		
Small	-0.58	-0.17	0.08	0.27	0.63	-(3.63)	-(1.41)	(0.72)	(2.42)	(4.31)
2	-0.15	0.03	0.23	0.30	0.41	-(1.17)	(0.37)	(2.74)	(3.37)	(3.54)
3	-0.15	0.10	0.20	0.30	0.34	-(1.79)	(1.36)	(2.80)	(4.06)	(3.06)
4	-0.06	0.02	0.16	0.19	0.32	-(0.96)	(0.42)	(2.82)	(2.60)	(3.09)
Big	-0.04	0.00	0.16	0.12	0.19	-(0.88)	-(0.06)	(2.82)	(1.75)	(2.06)
			$\hat{\beta}_i^m$					*t-stats for* $\hat{\beta}_i^m$		
Small	1.38	1.23	1.05	0.94	0.93	(41.44)	(53.78)	(61.31)	(53.78)	(44.14)
2	1.23	1.19	1.02	1.02	1.08	(55.19)	(143.99)	(67.92)	(62.38)	(58.57)
3	1.22	1.09	1.03	1.03	1.05	(87.35)	(78.21)	(79.60)	(80.43)	(54.60)
4	1.08	1.07	1.07	1.04	1.09	(87.25)	(125.88)	(116.27)	(66.75)	(54.52)
Big	0.98	0.94	0.91	0.97	1.03	(126.75)	(108.86)	(85.56)	(67.14)	(50.08)
			$\hat{\beta}_i^{UV}$					*t-stats for* $\hat{\beta}_i^{UV}$		
Small	-0.24	-0.29	-0.25	-0.31	-0.41	-(2.41)	-(3.80)	-(3.33)	-(6.22)	-(6.01)
2	-0.13	-0.09	-0.24	-0.24	-0.29	-(2.32)	-(2.28)	-(5.67)	-(5.31)	-(4.69)
3	-0.07	-0.19	-0.13	-0.11	-0.20	-(1.51)	-(5.04)	-(4.06)	-(3.61)	-(3.39)
4	-0.13	-0.08	-0.10	0.03	-0.11	-(3.65)	-(3.98)	-(3.51)	(1.05)	-(2.41)
Big	0.03	0.02	0.04	0.14	0.12	(1.32)	(0.65)	(1.43)	(4.74)	(2.53)

Table II - *Continued*

	Book-to-Market Equity (BE/ME) Quintiles									
Size Quintiles	*Low*	*2*	*3*	*4*	*High*	*Low*	*2*	*3*	*4*	*High*
			$\hat{\gamma}_{\mathrm{i}}$					*t-stats for* $\hat{\gamma}_{\mathrm{i}}$		
Small	-0.04	-0.07	-0.10	-0.11	-0.19	-(1.10)	-(2.07)	-(3.10)	-(3.47)	-(5.64)
2	-0.12	-0.07	-0.09	-0.09	-0.10	-(3.04)	-(1.94)	-(2.39)	-(2.56)	-(2.92)
3	-0.04	-0.05	-0.07	-0.03	-0.15	-(1.19)	-(1.28)	-(2.03)	-(0.90)	-(4.06)
4	-0.04	-0.08	-0.09	-0.07	-0.11	-(1.03)	-(2.31)	-(2.40)	-(1.97)	-(3.29)
Big	-0.10	0.00	-0.06	0.00	-0.02	-(3.24)	-(0.05)	-(1.47)	(0.02)	-(0.69)
			$\hat{\omega}_{\mathrm{i}}$					*t-stats for* $\hat{\omega}_{\mathrm{i}}$		
Small	0.43	1.09	0.17	0.19	0.02	(2.36)	(4.63)	(3.27)	(3.48)	(0.48)
2	3.35	0.17	0.63	0.20	0.25	(7.18)	(2.58)	(5.27)	(3.06)	(3.31)
3	0.21	0.15	0.16	0.25	0.14	(3.28)	(2.69)	(3.84)	(3.20)	(2.35)
4	0.22	0.10	0.15	0.18	0.26	(3.25)	(3.35)	(3.14)	(3.53)	(3.51)
Big	0.03	0.11	0.21	0.37	0.26	(2.30)	(3.60)	(4.29)	(7.85)	(4.24)
			$\hat{\theta}_{1,\mathrm{i}}$					*t-stats for* $\hat{\theta}_{1,\mathrm{i}}$		
Small	0.14	0.30	0.13	0.13	0.07	(12.88)	(12.02)	(9.56)	(9.20)	(15.96)
2	0.35	0.15	0.30	0.12	0.12	(12.08)	(7.96)	(12.08)	(8.17)	(8.90)
3	0.14	0.08	0.11	0.13	0.15	(7.77)	(5.09)	(6.37)	(7.34)	(11.81)
4	0.15	0.14	0.18	0.13	0.16	(7.73)	(6.99)	(6.46)	(10.43)	(10.38)
Big	0.05	0.12	0.14	0.17	0.13	(4.04)	(5.43)	(6.44)	(7.12)	(8.51)
			$\hat{\phi}_{1,\mathrm{i}}$					*t-stats for* $\hat{\phi}_{1,\mathrm{i}}$		
Small	0.87	0.71	0.88	0.87	0.94	(84.96)	(33.29)	(77.71)	(67.37)	(405.18)
2	0.49	0.85	0.67	0.87	0.88	(12.20)	(47.96)	(25.68)	(51.21)	(62.99)
3	0.85	0.89	0.86	0.84	0.86	(46.80)	(40.00)	(44.30)	(37.30)	(87.61)
4	0.80	0.84	0.79	0.86	0.84	(25.05)	(40.31)	(24.99)	(58.35)	(64.81)
Big	0.94	0.83	0.81	0.77	0.86	(59.65)	(28.46)	(27.68)	(32.22)	(58.69)

	Book-to-Market Equity (BE/ME) Quintiles				
Size Quintiles	*Low*	*2*	*3*	*4*	*High*
			R^2		
Small	0.51	0.53	0.63	0.62	0.55
2	0.71	0.76	0.76	0.75	0.69
3	0.81	0.86	0.84	0.80	0.72
4	0.87	0.90	0.86	0.81	0.72
Big	0.92	0.91	0.85	0.77	0.66

Table III
Stock Market: Risk Prices and Risk Premiums

Factors' risk prices in Panel A) are estimated for the period January 1927-December 2007 using the following asset pricing model:

$$E\left(R^e_{i,t+1}\right) = \lambda_0 + \lambda_m \beta_i^m + \lambda_{UV} \beta_i^{UV} ,$$

and the Fama-MacBeth (1973) procedure. The left-hand side variable is a vector of expected excess value-weighted returns for the 25 size- and value-sorted portfolios and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses. Volatility risk premiums in Panel B) are computed as the product between volatility risk price and volatility loadings. Results are reported in percentages, on a monthly basis.

Panel A) Factors' Risk Prices

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	R^2_{adj}
1.98	-1.10	-1.10	51.88
(5.41)	-(2.96)	-(3.07)	

Panel B) Volatility Risk Premiums

	Book-to-Market Equity (BE/ME) Quintiles					
Size Quintiles	*Low*	*2*	*3*	*4*	*High*	*High-Low*
Small	0.26	0.32	0.28	0.34	0.46	0.20
2	0.15	0.10	0.26	0.27	0.32	0.18
3	0.07	0.21	0.14	0.12	0.22	0.15
4	0.14	0.08	0.11	-0.03	0.13	-0.01
Big	-0.03	-0.02	-0.04	-0.16	-0.14	-0.11
Small-Big	0.29	0.34	0.32	0.49	0.59	

Table IV
Factors' Average Returns and Cross-Correlations

The R^e_m is the excess market return. The *MOM* is the momentum factor of Jegadeesh and Titman (1993). The *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. Data cover the period January 1927 to December 2007. **, * denote significance at the 1%, respectively 5% levels. The *t*-statistics are reported in parentheses.

Panel A) Average Returns for the Traded Factors

	R^e_m	*SMB*	*HML*	*MOM*
Average	0.68	0.24	0.42	0.75
t-stat	(3.92)	(2.21)	(3.67)	(5.14)

Panel B) Cross-Correlation Matrix with the 1st Unexpected Volatility Measure

	R^e_m	*UV*	*SMB*	*HML*	*MOM*
R^e_m	1.00				
UV	-0.33**	1.00			
SMB	0.34**	-0.18**	1.00		
HML	0.20**	0.31	0.09**	1.00	
MOM	-0.31**	-0.11**	-0.16**	-0.38**	1.00

Panel C) Cross-Correlation Matrix with the 2nd Unexpected Volatility Measure

	R^e_m	*UV*	*SMB*	*HML*	*MOM*
R^e_m	1.00				
UV	-0.35**	1.00			
SMB	0.34**	-0.22**	1.00		
HML	0.20**	0.06*	0.09**	1.00	
MOM	-0.31**	-0.07*	-0.16**	-0.38**	1.00

Table V
Stock Market: Risk Prices while Controlling for Other Risk Factors

Factors' risk prices for the period January 1927 to December 2007 are estimated using the Fama-MacBeth (1973) procedure for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right)=\lambda_0+\lambda_m\beta_i^m+\sum_{s=1}^{p}\lambda_s\beta_i^s \ ,$$

where λ_m is the price of market risk and λ_s is the price of risk associated with the generic factor *s*. The left-hand side variable is a vector of expected excess value-weighted returns for the 25 size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2_{adj}
1.46	-0.47					1.07
(3.53)	-(1.03)					
1.98	-1.10	-1.10				51.88
(5.41)	-(2.96)	-(3.07)				
1.23	-0.49	-0.95		0.52		82.17
(3.30)	-(1.16)	-(2.70)		(4.33)		
-0.13		-0.86		0.49		60.28
-(0.37)		-(2.10)		(4.19)		
1.15	-0.40	-1.08		0.53	0.63	82.63
(3.99)	-(1.27)	-(3.66)		(4.41)	(0.88)	
1.06	-0.33	-1.08	0.24	0.51		86.94
(2.99)	-(0.88)	-(3.06)	(1.89)	(4.49)		
1.20	-0.48	-1.12	0.23	0.50	-0.50	84.93
(3.08)	-(1.20)	-(3.01)	(1.81)	(4.51)	-(0.60)	
1.51	-0.79		0.19	0.48		77.43
(4.05)	-(1.95)		(1.44)	(4.22)		

Table VI
Stock Market: Risk Prices under Different Model Specifications

Factors' risk prices are estimated using the Fama-MacBeth (1973) procedure for the period January 1927 to December 2007 for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right)=\lambda_m\beta_i^m+\sum_{s=1}^{p}\lambda_s\beta_i^s \ ,$$

where λ_m is the price of market risk, and λ_s is the price of risk associated with the generic factor *s*. The left-hand side variable is a vector of expected excess value-weighted returns for the 25 size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2_{adj}
0.85					91.88
(4.09)					
0.75	-0.96				92.76
(3.94)	-(2.60)				
0.64	-0.63		0.51		97.59
(3.56)	-(1.66)		(4.22)		
	-0.75		0.48		97.71
	-(3.77)		(3.98)		
0.71	-1.26		0.56	1.73	98.28
(4.09)	-(4.14)		(4.63)	(2.25)	
0.70	-1.54	0.26	0.52		98.82
(4.01)	-(4.32)	(2.02)	(4.53)		
0.70	-1.51	0.24	0.52	0.31	98.58
(4.04)	-(3.87)	(1.92)	(4.65)	(0.40)	
0.68		0.19	0.48		97.74
(3.87)		(1.47)	(4.19)		

Table VII
Stock Market: Risk Prices for Different Samples

Factors' risk prices are estimated using the Fama-MacBeth (1973) procedure for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right) = \lambda_0 + \lambda_m \beta_i^m + \sum_{s=1}^{p} \lambda_s \beta_i^s ,$$

where λ_m is the price of market risk and λ_s is the price of risk associated with the generic factor s. Two additional sample periods are used. The left-hand side variable is a vector of expected excess value-weighted returns for the 25 size- and value-sorted portfolios, and the betas represent factors' loadings from the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). *PS* represents the Pastor and Stambaugh (2003) liquidity factor, available only for the sub-period 1963-2007. The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

Panel A) Post-depression sample (January 1935 to December 2007)

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2_{adj}
1.63	-0.66					7.92
(4.55)	-(1.66)					
2.02	-1.20	-1.06				68.12
(6.19)	-(3.51)	-(3.49)				
1.27	-0.53	-0.82		0.48		81.71
(4.30)	-(1.57)	-(2.89)		(4.42)		
-0.02		-0.77		0.50		65.60
-(0.06)		-(2.17)		(4.57)		
1.10	-0.36	-1.03		0.50	0.87	81.12
(4.69)	-(1.32)	-(4.61)		(4.57)	(1.33)	
1.08	-0.35	-0.87	0.18	0.49		81.64
(3.59)	-(1.06)	-(3.01)	(1.65)	(4.66)		
1.11	-0.39	-0.95	0.18	0.48	-0.27	81.16
(3.69)	-(1.17)	-(2.87)	(1.66)	(4.60)	-(0.47)	
1.52	-0.80		0.15	0.47		75.29
(4.98)	-(2.32)		(1.34)	(4.49)		

Table VII - *Continued*

Panel B) Post-COMPUSTAT sample (January 1963 to December 2007)

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	$\hat{\lambda}_{PS}$	R^2_{adj}
1.57	-0.79						18.28
(3.62)	-(1.63)						
1.91	-1.31	-0.76					78.03
(4.76)	-(3.04)	-(2.72)					
2.18	-1.40	-0.88				-0.02	78.61
(5.89)	-(4.40)	-(2.91)				-(1.21)	
1.33	-0.76	-0.73		0.42			78.76
(4.51)	-(2.21)	-(2.62)		(3.11)			
0.10		-0.83		0.40			59.14
(0.39)		-(2.69)		(3.05)			
1.07	-0.50	-0.91		0.46	1.76		81.03
(3.43)	-(1.41)	-(3.75)		(3.26)	(2.43)		
1.30	-0.75	-0.68	0.20	0.45			78.10
(3.99)	-(1.96)	-(1.98)	(1.38)	(3.47)			
1.28	-0.73	-0.70	0.20	0.46	0.50		77.28
(3.84)	-(1.87)	-(2.05)	(1.40)	(3.57)	(0.71)		
1.35	-0.81		0.18	0.46			74.75
(4.14)	-(2.10)		(1.23)	(3.51)			

Table VIII

Treasury Market: Average Excess Returns, Factor Loadings, Risk Prices and Risk Premiums

Monthly average excess returns for Fama bond portfolios are reported in percentages in Panel A). Factors' loadings are reported in Panel B). Factors' risk prices in Panel C) are estimated for the period January 1952 to December 2007 in the following asset pricing model:

$$E\left(R^{e}_{i,t+1}\right)=\lambda_m \beta_i^{m}+\lambda_{UV}\beta_i^{UV},$$

using the Fama-MacBeth (1973) procedure. The left-hand side variable is the vector of excess equal-weighted returns on the 12 Fama maturity-sorted bond portfolios. Betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. Risk premiums in Panel D) are estimated as the product between factors' risk price and factors' loadings. Results are reported in percentages, on a monthly basis. The *t*-statistics are reported in parentheses. They include the Shanken (1992) correction in Panel C).

Panel A) Average Excess Returns

Maturity

1-6 mo	*7-12 mo*	*13-18 mo*	*19-24 mo*	*25-30 mo*	*31-36 mo*	*37-42 mo*	*43-48 mo*	*49-54 mo*	*55-60 mo*	*61-120 mo*	*>120 mo*
0.04	0.06	0.08	0.09	0.10	0.11	0.12	0.12	0.13	0.09	0.13	0.19
(7.13)	(4.49)	(3.90)	(3.20)	(2.95)	(2.82)	(2.69)	(2.52)	(2.48)	(1.66)	(1.98)	(1.78)

Panel B) Factors' Loadings

Maturity

1-6 mo	*7-12 mo*	*13-18 mo*	*19-24 mo*	*25-30 mo*	*31-36 mo*	*37-42 mo*	*43-48 mo*	*49-54 mo*	*55-60 mo*	*61-120 mo*	*>120 mo*
					$\hat{\beta}_{\mathrm{i}}^{\mathrm{m}}$						
0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.03	0.05	0.03	0.09
(2.83)	(3.36)	(3.19)	(4.24)	(4.03)	(4.03)	(3.67)	(4.03)	(3.34)	(4.88)	(2.70)	(4.41)
					$\hat{\beta}_i^{UV}$						
0.00	0.02	0.04	0.06	0.07	0.07	0.09	0.10	0.09	0.12	0.10	0.19
(4.65)	(5.17)	(4.71)	(5.13)	(4.40)	(3.90)	(4.00)	(3.67)	(3.01)	(3.84)	(2.63)	(3.11)

Panel C) Factors' Risk Prices

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	R^2_{adj}
0.04	-2.02	1.49	85.73
(4.00)	-(1.69)	(1.86)	
	-4.07	2.64	96.59
	-(2.88)	(3.27)	

Panel D) Volatility Risk Premiums

Maturity

1-6 mo	*7-12 mo*	*13-18 mo*	*19-24 mo*	*25-30 mo*	*31-36 mo*	*37-42 mo*	*43-48 mo*	*49-54 mo*	*55-60 mo*	*61-120 mo*	*>120 mo*
0.00	0.03	0.06	0.09	0.10	0.10	0.13	0.15	0.13	0.18	0.15	0.28

Table IX

Risk Prices in the Joint Markets

Factors' risk prices for the period January 1952 to December 2007 are estimated using the Fama-MacBeth (1973) procedure for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right) = \lambda_0 + \lambda_m \beta_i^m + \sum_{s=1}^{p} \lambda_s \beta_i^s \ ,$$

where λ_m is the price of market risk and λ_s is the price of risk associated with the generic factor *s*. The left-hand side variable is a vector of excess returns for 12 maturity-sorted Treasury portfolios and 25 size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2_{adj}
0.54	0.36					74.27
(1.93)	(1.07)					
0.79	0.01	-1.02				78.22
(3.07)	(0.03)	-(2.85)				
0.37	0.30	-0.72		0.51		92.77
(1.25)	(0.86)	-(1.97)		(4.27)		
-0.07		-0.80		0.48		91.82
-(0.26)		-(2.43)		(4.00)		
0.50	0.23	-1.16		0.54	1.25	94.62
(2.23)	(0.89)	-(4.02)		(4.51)	(1.67)	
0.42	0.29	-1.19	0.24	0.51		95.54
(1.47)	(0.96)	-(3.51)	(1.89)	(4.47)		
0.43	0.28	-1.14	0.23	0.52	0.64	95.20
(1.32)	(0.82)	-(3.30)	(1.82)	(4.63)	(0.78)	
0.46	0.24		0.19	0.48		92.98
(1.51)	(0.69)		(1.46)	(4.20)		

Table X
Posterior Estimates for the *Asymmetric Student-GARCH(1,1)* Model

The model used is:

$$\begin{cases} \mathrm{R}_{\mathrm{m,t}} = \mu + \rho \mathrm{R}_{\mathrm{m,t-1}} + \eta_{\mathrm{t}} \\ \eta_{\mathrm{t}} = \varepsilon_{\mathrm{t}} \sqrt{\mathrm{h}_{\mathrm{t}}} \\ \varepsilon_{\mathrm{t}} / \mathrm{I}_{\mathrm{t-1}} \sim \mathrm{Student}(0,1,\upsilon) \\ \mathrm{h}_{\mathrm{t}} = \alpha + \phi \mathrm{h}_{\mathrm{t-1}} + \theta^{+} \eta_{\mathrm{t-1}}^{2+} + \theta^{-} \eta_{\mathrm{t-1}}^{2-} \\ \eta_{\mathrm{t}}^{2+} = \eta_{\mathrm{t}}^{2} 1_{\{\eta_t > 0\}}, \eta_{\mathrm{t}}^{2-} = \eta_{\mathrm{t}}^{2} 1_{\{\eta_t < 0\}} \end{cases} \quad , t=1,\dots,T.$$

The R_t represents the monthly time series of returns on the stock market portfolio for the period January 1927 – December 2007. The results are computed using a Griddy-Gibbs sampling algorithm in which 5000 draws are kept and 1000 initial draws are the burn-in sample. A flat prior on finite intervals is used on all parameters except for the prior on υ, that is half-Cauchy. Standard errors are reported in the parentheses.

	Estimate	*Std Err*
μ	0.20	(0.03)
ρ	0.05	(0.03)
α	0.06	(0.03)
θ^{+}	0.05	(0.04)
ϕ	0.75	(0.07)
υ	8.61	(2.51)
θ^{-}	0.20	(0.06)

Table XI
Stock Market: Robustness with Respect to an Alternative Volatility Measure

Factors' risk prices are estimated using the Fama-MacBeth (1973) procedure for the period January 1927 to December 2007 for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right) = \lambda_0 + \lambda_m \beta_i^m + \sum_{s=1}^{p} \lambda_s \beta_i^s \ ,$$

where λ_m is the price of market risk and λ_s is the price of risk associated with the generic factor *s*. The left-hand side variable is a vector of expected excess value-weighted returns for the size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and it is constructed as the difference between predicted and realized volatility (see equations (16) and (4)). *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2_{adj}
1.46	-0.47					1.07
(3.53)	-(1.03)					
2.12	-1.24	-0.86				45.97
(5.86)	-(3.42)	-(2.63)				
1.28	-0.54	-0.74		0.51		80.44
(3.34)	-(1.28)	-(2.20)		(4.28)		
0.10		-0.59		0.50		65.49
(0.37)		-(2.11)		(4.23)		
1.15	-0.40	-0.89		0.52	0.65	80.09
(3.93)	-(1.27)	(2.67)		(4.37)	(0.85)	
1.13	-0.40	-1.12	0.23	0.50		85.64
(3.26)	-(1.10)	-(2.88)	(1.81)	(4.39)		
1.05	-0.35	-1.38	0.22	0.48	-1.27	83.19
(2.98)	-(0.96)	-(2.94)	(1.77)	(4.19)	-(1.20)	
1.51	-0.79		0.19	0.48		77.43
(4.05)	-(1.95)		(1.44)	(4.22)		

Table XII
Predictive Variables: Descriptive Statistics and Contemporaneous Relations with Volatility

Descriptive statistics for the actual predictors are presented in Panel A). Panel B) documents the contemporaneous relations between shocks to predictors and volatility using a time series model. The shocks are computed as the residuals from a first-order VAR system. In Panel B) *DEF* represents shocks to the default spread (the default spread itself is computed as the difference between the yields of a long-term corporate Baa bond and a long-term corporate bond Aaa), *TERM* represents shocks to the term spread (the term spread itself is computed as the difference between the yields of a thirty-year and a one-year government bond), *DY* represents shocks to the dividend yield on the S&P500 value-weighted portfolio (the dividend yield itself is computed as the sum of dividends over the last 12 months, divided by the level of the index), and R_f represents shocks to the one-month T-bill yield. Data cover the period January 1942 to December 2007. The *t*-statistics are reported in parentheses. The *F*-test with its corresponding p-value for the joint significance of the factor estimates is reported in the last column.

Panel A) Descriptive Statistics for Predictors

	Def	*TERM*	*DY*	R_f
Avg	0.91	0.04	3.71	0.35
Std Dev	0.40	2.34	1.48	0.25
Skewness	1.50	0.14	0.48	0.96
Kurtosis	2.61	1.90	0.09	1.24

Panel B) Contemporaneous Relations between Shocks to Predictors and Volatility

$\hat{\beta}_0$	$\hat{\beta}_m$	$\hat{\beta}_{DEF}$	$\hat{\beta}_{TERM}$	$\hat{\beta}_{DY}$	$\hat{\beta}_{Rf}$	R^2_{adj}	F
-0.07	-0.16	0.17	0.04	-0.87	2.62	14.74	28.38
-(1.47)	-(8.41)	(0.30)	(2.09)	-(2.10)	(3.16)		(<0.0001)

Table XIII
Stock Market: Horse-Race Tests with Predictive Variables

Factors' risk prices for the period January 1942 to December 2007 are estimated using the Fama-MacBeth (1973) procedure for the following asset pricing model:

$$E\left(R_{i,t+1}^{e}\right)=\lambda_0+\lambda_m\beta_i^m+\sum_{s=1}^{p}\lambda_s\beta_i^s \ ,$$

where λ_m is the price of market risk and λ_s is the price of risk associated with the generic factor s. The left-hand side variable is a vector of expected excess value-weighted returns for the size- and value-sorted portfolios, and the betas represent factors' loadings estimated in the corresponding time series models. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the time series of realized stock market volatility. *MOM* is the momentum factor of Jegadeesh and Titman (1993). *HML* and *SMB* are mimicking portfolios for book-to-market equity and size (zero-investment portfolios). *DEF* represents shocks to the default spread (the default spread itself is computed as the difference between the yields of a long-term corporate Baa bond and a long-term corporate bond Aaa), *TERM* represents shocks to the term spread (the term spread itself is computed as the difference between the yields of a thirty-year and a one-year government bond), *DY* represents shocks to the dividend yield on the S&P500 value-weighted portfolio (the dividend yield itself is computed as the sum of dividends over the last 12 months, divided by the level of the index), and R_f represents shocks to the one-month T-bill yield. The *t*-statistics (with the Shanken (1992) correction) are reported in parentheses.

$\hat{\lambda}_0$	$\hat{\lambda}_m$	$\hat{\lambda}_{UV}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{DEF}$	$\hat{\lambda}_{TERM}$	$\hat{\lambda}_{DY}$	$\hat{\lambda}_{Rf}$	R^2_{adj}
2.20	-1.41			-0.10	0.21	0.07	-0.12	63.04
(7.25)	-(4.24)			-(3.31)	(0.41)	(1.87)	-(4.78)	
1.59	-0.85	-1.17		-0.03	0.69	0.00	-0.04	76.47
(5.20)	-(2.52)	-(4.23)		-(0.99)	(1.30)	(0.02)	-(2.17)	
1.17	-0.45	-0.64	0.46	-0.02	0.30	0.03	-0.02	81.77
(3.99)	-(1.40)	-(2.81)	(4.47)	-(1.13)	(0.56)	(0.89)	-(1.34)	
1.22	-0.50	-0.68	0.47					82.08
(4.12)	-(1.54)	-(2.67)	(4.61)					

Table XIV
Volatility and Shocks to Cash Flows

The volatility effects on cash flows for the period January 1962 to December 2007 are estimated in a time series model for the market equity and book-to-market equity sorted portfolios. *UV* represents unexpected volatility, and is the residual from an AR(1) model applied to the quarterly time series of realized stock market volatility. The left-hand side variable is a vector of shocks to the quarterly average portfolio cash flows. To reduce the impact of outliers on the results, I Winsorize the cash flow data at 1% and 99%. The *t*-statistics are reported in parentheses.

	Size-Sorted Portfolios				*Book-to-Market Sorted Portfolios*		
	$\hat{\beta}_0$	$\hat{\beta}_{UV}$	R^2_{adj}		$\hat{\beta}_0$	$\hat{\beta}_{UV}$	R^2_{adj}
Small	-0.06	-0.31	2.00	*Growth*	-0.01	-0.03	0.00
	-(0.17)	-(2.17)			-(0.08)	-(1.00)	
Medium	-0.00	-0.01	-0.31	*Mid-Value*	-0.00	-0.01	-0.16
	-(0.05)	-(0.68)			-(0.06)	-(0.86)	
Big	0.00	0.01	-0.36	*Value*	-0.01	-0.04	3.94
	(0.05)	(0.60)			-(0.24)	-(2.90)	